On Applications of the Retracing Method for Distance-Regular Graph

A Thesis
Presented to
The Faculty of the Mathematics Department
College of Science
De La Salle University - Manila

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Mathematics

by
Norma Luz C. Vencer

November 2006

Abstract

This thesis is an exposition of the article written by Akira Hiraki entitled Applications of Retracing Method for Distance-Regular Graphs published in European Journal of Combinatorics, April 2004. The main results of the article are as follows:

Theorem 1.1 Let Γ be a distance-regular graph of diameter d with

$$r = |\{\ i\ |(c_i, a_i, b_i) = (c_1, a_1, b_1)\}|\ \geq 2$$

and $c_{r+1} \geq 2$. Let m, s and t be positive integers with $s \leq m$, $m + t \leq d$ and (s, t) $\neq$ (1,1). Suppose $b_{m-s+1} = \cdots = b_m = 1 + b_{m+1}$, $c_{m+1} = \cdots = c_{m+t} = 1 + c_m$ and $a_{m-s+2} = \cdots = a_{m+t-1} = 0$. Then the following hold.

(1) If $b_{m+1} \geq 2$, then $t \leq r - 2\lfloor s/3 \rfloor$.

(2) If $c_m \geq 2$, then $s \leq r - 2\lfloor t/3 \rfloor$.

Corollary 1.2. Under the assumption of Theorem 1.1, the following hold.

(1) If $r = t$ and $b_{m+1} \geq 2$, then $s \leq 2$.

(2) If $r = s$ and $c_m \geq 2$, then $t \leq 2$.

Corollary 1.3. Let Γ be a distance-regular graph of valency $k \geq 3$ with

$c_1 = \cdots = c_r = 1$, $c_{r+1} = \cdots = c_{r+t} = 2$ and $a_1 = \cdots = a_{r+t-1} = 0$.

(1) If $k \geq 4$, then $t \leq r - 2\lfloor r/3 \rfloor$.

(2) If $2 \leq t = r$, then Γ is either the Odd graph, or the doubled Odd graph.

(3) If $2 \leq t = r - 1$, then Γ is the Foster graph.