INTEGRATED TEACHING OF ALGEBRA AND GEOMETRY IN SECONDARY SCHOOLS

A Thesis

Presented to

the Faculty of the School of Graduate Studies

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In Partial Fulfillment

of the Requirements for the Degree

Master of Arts in Education

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by

Nenita P. Sornito

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A THESIS

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Approved:

Adviser

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In undertaking this experimental project, the writer was aware of the difficulties involved. She recognized that she needed the advice and assistance of men and women with mature judgment and recognized teaching experience, in order to accomplish her objective, which is to contribute to the upgrading of the teaching of mathematics in the high school.

While she is undoubtedly happy to present this finished work, she humbly acknowledges the invaluable help extended to her by the following persons:

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With the unselfish assistance of such dedicated members of the faculty of the School of Graduate Studies of Central Philippine University, it is earnestly hoped that this humble work will fulfill the writer's desire to contribute to the improvement of the teaching of mathematics in the secondary schools.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The problem of upgrading instruction is a pressing problem for all educational institutions everywhere. The advance of civilization, the ever-increasing human needs and the changing mode of living demand that the education of the masses should be continuously upgraded in order to keep pace with the march of progress. This is particularly true in this century when increase in learning and new discoveries in science have attained an acceleration heretofore unparalleled in the history of mankind. The youth of today have much more to learn than their fathers had, and their children will have to cope with knowledge and concepts that would make the present civilization look primitive.

In the light of these changes, educational institutions are faced with a tremendous problem--that of upgrading the methods and patterns of instruction in order to increase the learning efficiency of the youth. To meet this challenge, institutions of learning, particularly those devoted to higher education, are channeling a portion of their resources to the search for more effective instructional patterns. One area of this research is concerned with the upgrading of instruction in mathematics.

To complicate matters for Philippine education, the tendency in the years just past was a retrogression in achievement that has set back the present school children. In a brochure prepared by the Secondary Education Section, Instruction Division, Bureau of Public Schools, in cooperation with Francis H. Vittetow, Education Advisor (Elementary-Secondary), Agency for International Development, mention was made of this:

> The 1960 Swanson Survey has found that the pupils of elementary arithmetic now are two grades below and the high school students of arithmetic are two-and-a-half years behind their 1925 counterparts in arithmetic computation and language and in arithmetic reasoning, respectively. These findings are indicative of a need for reexamining our program of mathematics offerings, upgrading the competency of mathematics teachers and improving the materials and techniques of mathematics instruction.[1]

This thesis presents to educators the results obtained from a comparative study of two methods: a "new" pattern of teaching mathematics in contrast to the usual method or pattern of instruction. For want of a name, this pattern is called "Integrated Teaching of Algebra and Geometry in Secondary Schools." Some educators call it "Unified Mathematics," others call it "General Mathematics," "Basic Mathematics," "Concentrated Mathematics," and many other names. All imply the same principle and concept. It envisions a course in which algebra and geometry are fused into one single subject

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[1] Jose T. Cortes /Assistant Chief (In-Charge)/, _Mathematics in the High School_ (Manila: Bureau of Public Schools, 1962).

instead of the compartmentalized studies that we now have. This is not an entirely new idea. Many mathematics teachers have made casual mention of this and expressed favorable opinion regarding its value and merit. But, as yet, no adoption of this pattern of instruction in this country is on record. In the words of Professor Reeve, "We have not yet been able to develop a basic course in real mathematics.[2]

In undertaking this project, the writer realized the difficulties involved. She was aware that a total integration of algebra and geometry is not possible. This opinion is concurred in by Professor Fehr in his report at the 13th Annual Meeting of the National Council of Teachers in Mathematics, when he said that in order to fuse these two subjects into a single structure, "we must recognize that a new set of postulates must be established, for surely those of Euclidean geometry are distinct and based on different elements than those of abstract algebra."[3] At present, there are no such set of postulates that the writer is aware of.

In the same annual meeting, Professor Meserve expressed an opinion in these words:

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[2] William D. Reeve, "The Need for a New National Policy and Progress in Secondary Mathematics," The Mathematics Teacher, 48:4, January, 1955.

[3] Howard F. Fehr, "Using Algebra in Teaching Geometry," The Mathematics Teacher, 45:561, December, 1952.

> Algebraic concepts may be used in defining elements of geometry, and geometric concepts may be used in defining elements of algebra . . . they indicate how the interdependence of algebra and geometry may be used to improve the presentation and understanding of both subjects in terms of fundamental concepts of mathematics.[4]

In teaching after the new pattern, the writer-teacher did not attempt to effect a complete fusion, but rather to use algebra and geometry to help each other by creating an interplay in the teaching of mathematics. The experimenter only tried to effect the integration in the teaching and study of fundamental concepts. Previous experience taught the futility of effecting complete fusion, particularly in the teaching of the higher manipulative operations in algebra and in the teaching of demonstrative geometry. Where there are cases in which fusion was practical and useful, integration was applied. The outline of the course as given in Appendix C[5] will show the areas of integration.

The nature of this project required that the experimenter considers and makes some personal adjustments if need be, in order to conform with the basic principles of research. This is necessary if the results obtained and the findings made are to be valid and reliable. For this purpose she was careful to observe the following guiding principles:

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[4] B. E. Meserve, "Using Geometry in Teaching Algebra," _The Mathematics Teacher_, 45:567, December, 1952.

[5] _Infra_, pp. 97-99.

1. Curiosity about environment and situation. The author kept herself mentally alert and curious about any developments in the experiment.

2. Belief in the principle that every effect has a natural cause.

3. Openmindedness or the readiness to take notice of and consider developments regardless of whether they agree or disagree with the working hypothesis.

4. Critical thinking or the willingness to look at situations from all angles.

5. Unwillingness to accept as facts, observations not supported by convincing proofs.

6. Readiness to change beliefs upon presentation of new contrary but reliable evidence.

7. Impartiality or the refusal to allow previous beliefs, prejudices and bias to influence judgment.

8. Respect for the viewpoints of others.

9. Maintenance of such ideals as honesty, patience, perseverance, fairness, and thoroughness throughout the experiment.

I. THE PROBLEM

General problem. The first concern of the government at the close of World War II was the accommodation in the schools of the fast-growing school population. To meet the situation, temporary classrooms had to be provided, equipment and other facilities had to be improvised and unqualified teachers had to be employed in the very delicate task of training the young people. As a natural consequence of this malpractice, the standard of instruction went on a "dive." On top of this, students were in such a hurry to graduate that they evaded mathematics which was generally considered a difficult subject and, therefore, a stumbling block to graduation. To make matters worse, the curriculum was watered down by the reduction of the minimum mathematics requirement. As a result of all these factors, students graduating from the high school have become unprepared for college work. This situation is strongly pronounced in the technical courses which require a strong background in mathematics.[6] "Quality" has evidently been sacrificed to attain "quantity."

It is comforting to note that it did not take long for the country to awaken to this dangerous educational trend. During the last decade a vigorous program of upgrading instruction has been launched in different areas of instruction. The education leaders of the Philippines have set to work to arrest the downhill trend. They have become convinced of the need for renovation of practices in education.

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[6] Leonard G. Yoder, "Mathematics and the Technical Manpower Shortage," The Mathematics Teacher, 48:550, December, 1955.

Among the different areas on which public attention was focused is science education. Perhaps no other area has suffered so much in the "educational black-out" during the post-war period, than this phase of instruction. Recognizing this fact, a special government entity, the National Science Development Board, was created to tackle the problem.

A precedent condition to the effective teaching and study of science is a strong background in mathematics. It is a matter of common knowledge that science instruction can not go far beyond its starting point, unless it is powered by mastery of the basic mathematical concepts; without such mastery, its structure would rest only upon a shaky foundation. It is a sad fact to note that one of the weakest programs, if not the weakest, in the present instruction is the teaching of this subject. This is largely due to the fact that students have developed a general aversion to mathematics, and consequently, teachers find it difficult to teach. The discovery of a more fruitful approach in mathematics instruction is, therefore, the general problem for which the writer sought to contribute an adequate proposal.

Specific problem. Work on the problem of upgrading instruction in any subject opens up several areas for study and exploration. Among them are the following:

1. The teacher-centered area. The problems here involve, in general, the qualifications and the personal qualities

of the teacher--his fitness for the job of teaching his subject and his devotion to it.

2. The curriculum-centered area. The problems here refer to the general programming of the course from the grade in which mathematics is first introduced as an instructional unit, to the grade or year when it is last taught. Also pertinent here are the sequential order of courses and the level of difficulty to be treated in the different grades.

3. The pupil-centered area. The problems here deal with the study of the students collectively or individually--their background, their fitness for the work prescribed, their personal traits, their intelligence, individual aims and objectives, and so forth, all in relation to the nature and character of the subject that is to be taught to them. Here, too, is the field of personal counseling.

4. The classroom or teaching-centered area. This area involves the actual contact between the teacher and the students--the act of motivation and guidance on the part of the teacher and the corresponding response and learning on the part of the students. It includes among other things the selection of teaching methods to be adopted by the teacher and his specific techniques, and his power to motivate and to hold the interest of the students in the subject he is teaching.

The upgrading of instruction consists in the proper solution of the problems involved in those areas and the proper implementation of whatever policy or pattern may be adopted. Each of the problems enumerated above can be a separate subject of research. In other words, any one of these areas can be studied independently. In this work, the author limited herself to the classroom-centered area, studying specifically the effectiveness of a certain organizational pattern in the teaching of the subject. The specific aim, therefore, is to test a pattern of instruction which is described above as "The Integrated Teaching of Algebra and Geometry in Secondary Schools." In short, the writer sought an answer to the question, "Is the integrated pattern of teaching algebra and geometry superior to the traditional pattern presently followed in our high schools?" This testing was further limited to its application in the study of basic concepts, leaving out the study of more complex algebraic operations and demonstrative geometry.

II. IMPORTANCE OF THE PROBLEM

The role of mathematics in present-day education. The role that mathematics plays in the education of young people cannot be overestimated. Civilization has so developed that what is ordinary and common place at this time was fiction half a century ago. In all present-day human activities, evaluation and rationalization have become necessary to the pattern of man's thinking. Studies of all sorts do not end with qualitative findings. They can be considered complete only after a thorough quantitative analysis. Knowledge in any subject is of little value in present technology unless that knowledge includes evaluation; that is, its "how much" is determined. The atomic and nuclear concepts were developed as early as the middle of the eighteenth century. But it was not until recently, when scientists have determined the weight and velocity of the component parts of the atoms and the nuclei, that atomic and nuclear power has become the servant of man. In another case, the use of numbers to count man's experience is as old as history itself. But it was not until the science of statistics was developed, or shall one say perfected, that reliable sociological measurements based on the laws of probability and prediction have become available for man's use. There is indeed some truth in the statement, "If one knows only the what, the how, and the why of a matter or force, he knows nothing about it, but if one knows the how much besides, then he begins to know something about it."

In order to realize fully the importance of the problem for which the investigator seeks to offer a solution, it is necessary to mention here the basic aims of the teaching of mathematics. The American National Committee on Mathematical Requirements prepared the following generalized list:[7]

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[7] David R. Davis, The Teaching of Mathematics (Reading, Massachusetts: Addison-Wesley Publishing Co., Inc., 1951), p. 2.

1. Practical aims--The cultivation of an understanding of the fundamental concepts and processes of mathematics sufficient to perform efficiently the vocational tasks required of the individual.

2. Disciplinary aims--The development of the power to think logically, to critically analyze a given situation, to determine relative values, and to reach definite conclusions which can be substantiated.

3. Cultural aims--The acquisition of an appreciation of mathematics for its precision, beauty, power, systematic organization, clarity of symbolic language, exact logical reasoning, and its great capacity for yielding generalizations and prediction.

The general objectives for the teaching of mathematics in the Philippine Schools are the following:[8]

1. Proficiency in fundamental skills.
2. Comprehension of basic concepts.
3. Appreciation of significant meanings.
4. Development of desirable attitudes.
5. Efficiency in making sound applications.
6. Confidence in making intelligent and independent interpretation.

Much can be said and written about the aims enumerated above, but it is beyond the scope of this work which concerned itself with the study of the relative merits of two contrastive patterns of instruction to be used in the attainment of these objectives. It is hoped that this experiment can indicate which pattern of instruction is the more promising in the accomplishment of the aims mentioned above.

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[8] Cortes, loc. cit.

III. DEFINITIONS OF TERMS USED

For the purpose of this paper, a few terms need to be defined. The definitions given below will be the meaning intended by the author in the use of the terms, although other writers may attach slightly different meanings to them in their works.

Upgrading. Upgrading as used here will mean the increase of teaching efficiency to the end that more and lasting learning is effected.

Integrated mathematics. Integrated mathematics (Also Unified Mathematics, General Mathematics, Concentrated Mathematics, or Correlated Mathematics), will mean the fusion of arithmetic, algebra, and geometry into a single mathematical structure so that they are taught in a fashion that makes them the means instead of the objective in teaching a principle or a concept. "It means a complete course organized in such a way as to show how the various subjects are related in order that they may reinforce and supplement each other in useful ways."[9]

Algebra. It is a branch of mathematics which deals with generalized values (variables), as opposed to Arithmetic which

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[9] William D. Reeve, "General Mathematics in the Secondary Schools," The Mathematics Teacher, 47:77, February, 1954.

deals with specific values (constants). It is to be understood that the two belong to the same area of mathematics in that they both deal with "quantity" or "count," by the use of symbols. It is said that algebra is just "adult arithmetic" and arithmetic is just "infant algebra."

Geometry. Geometry as understood here means the science of space in zero dimension (point); in one dimension (line); in two dimensions (surface); and in three dimensions (solid) (Euclidean Geometry).

Traditional mathematics. Traditional mathematics as used here refers to the so-called "sequential," "compartmentalized" mathematics as one generally teaches it in the Philippine schools today. It means the teaching of arithmetic, algebra, and geometry, as distinct subjects in the different grades, without showing any relation that obtains among them.

Experimenter. It refers to the writer of this thesis.

All other terms used elsewhere in this thesis will be used according to their usual meaning in common usage.

IV. GENERAL SCHEME AND DESIGN

As mentioned in the introduction, this is an experimental project in which the procedural pattern characteristic of all experimental research was followed. The steps taken may be briefly stated here and later will be discussed at length in the general procedure.

1. The problem was isolated into a hypothesis which was to be tested by the research.

2. The factors involved were defined and special efforts were put forth to equate them, with the exception of one variable which was the subject of the research.

3. Two comparable classes or groups which were to be used in the experiment were created and balanced; that is, the selection was done so as to suppress the effect of variables which were not under study. In order to do this a survey test was conducted and membership and the classes were equated on the basis of the results.

4. One method of teaching (the traditional pattern) was consistently used with one class; the other (the integrated pattern), with the other class.

5. At the end of the term a common test was administered to both to determine the relative achievements of the two groups.

6. The results of the tests were statistically studied, analyzed, and evaluated.

7. A summary of the observations and findings, including the result of the statistical analysis, was drawn.

8. Conclusions and final recommendations were based upon the summary and findings in (7).

CHAPTER II

REVIEW OF RELATED READINGS AND LITERATURE

On the educational value of mathematics. Much has been said about the general educational value of mathematics, both as a tool and as a discipline. Professor John R. Abernethy summarized these values, thus:

> Mathematics in general education rests upon three axioms. We believe: (1) that mathematics contains values that everyone needs; (2) that most people are capable of acquiring needed values from mathematics; and (3) that the worth of the individual justifies the effort required to give him or her these values.[1]

He further enumerated these values as:

1. The general value of mathematics as basic factual information and skills that make it a tool for everybody, that make it a key to other knowledge, and that make mathematics the queen of the sciences because it is the servant of all. Through the use of mathematics as a tool, science has brought comfort to mankind and made possible increased production for the satisfaction of human wants.

2. The general educational value as an aid to personal adjustment. It is generally admitted that mathematics is a tough subject. But one who submits to the rigid discipline of mathematics learns to adjust himself to the "difficult" in life.

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[1] John R. Abernethy, "General Educational Values of Mathematics and the Attempt of a Faculty to Teach Them," The Mathematics Teacher, 46:241, April, 1953.

He develops self confidence and poise, and gets over his timidity and inferiority complex.

3. The general educational value of the experience it offers in unprejudiced and logical thinking. It is the only field of endeavor that operates without prejudice, in favoring neither race, creed, economic position, or form of government. It can be practised without any emotional disturbance. It is the science that opens the broadest field to international understanding. The facts of mathematics are the same for all people regardless of their color, culture or way of life.

4. The fourth general educational value of aesthetics. It gives meaning to harmony, symmetry, order, beauty, elegance, and everything that appeals to the noblest in man. From *Mathematics in Aristotle*, one can read:

> For the chiefest forms of the beautiful are orderly arrangement, symmetry, and definiteness and the mathematical sciences have these characters in the highest degree. And since these characters, such as orderly arrangement and definiteness, are the causes of many things, it is clear that mathematicians could claim that this sort of cause is in a sense like the beautiful acting as a cause.[2]

5. The general educational value of the insight it gives one into the knowledge and understanding of the Filipino cultural heritage. Cassius Jackson Keyser emphasized this in his book, *Mathematics as a Culture Cue*.[3] And while mathematics cannot take over the task of teaching religion, the beginning

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[2] *Ibid.*, p. 242.

[3] *Ibid.*, pp. 242-243.

of mathematics is God, because back of the creation is the design, and the design is mathematical.[4]

Mathematics has a universal character. It was Sidney Hacker who said:

> It is peculiar among all the sciences in that it does not lend itself in any way to popular exposition. There are no gadgets to display with a glib veneer of "explanation" on how they work. There are no experiments to perform magician-like in order to amuse or to entertain. Mathematics at any level requires individual careful thinking, which means hard work for most people and is downright distasteful to many. In order to understand and appreciate mathematics one has got to make up his mind to think. There is no alternative.[5]

On the status of the present-day mathematics instruction. In the last few years, many articles have appeared, particularly in mathematics journals, deploring the poor quality of mathematics instruction in secondary schools. No less than the late President Magsaysay himself deplored the weakness of the science and mathematics programs of the secondary schools and, because of this, expressed his misgivings over the future

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[4] William T. Meyer, "Religion in Higher Education," Current Issues in Higher Education, 1950 (Report of the 5th Annual National Conference on Higher Education [Washington, D.C.: N.E.A., Department of Higher Education, 1951], pp. 113-18; as cited by John R. Abernethy, "General Educational Values of Mathematics and the Attempt of a Faculty to Teach Them," The Mathematics Teacher, 46:243, April, 1953.

[5] Sidney Hacker, Arithmetical Viewpoints (1948), p. iii; as cited by William L. Schaaf (ed.), "Memorabilia Mathematica," The Mathematics Teacher, 50:70, January, 1957.

economy and industrialization projects of the government. The 1960 Swanson Survey mentioned in Chapter I also revealed the sad state of Filipino mathematics instruction in this country.[6] This is true not only in the Philippines but also in more advanced countries like the United States.

The decline of mathematics instruction in the Philippine schools is due to a number of causes, chief among them are the following:

1. The depletion of the supply of competent mathematics teachers. It is not that one lacks mathematical talents. There are certainly many well qualified for the job.[7] But, comparatively speaking, the compensation is so low that they are drawn to industry and other pursuits where the material returns are more attractive. Only a few who teach the subject for the love of it or who cannot find a more remunerative position stay on the job. This is one of the chief problems of the Philippine schools--the problem of keeping competent teachers in the service.[8]

Dr. Aldana[9] adds a new twist to the competition. One

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6 Jose T. Cortes [Assistant Chief (In-Charge)], Mathematics in the High School (Manila: Bureau of Public Schools, 1962).

7 Ray C. Maul, "Where Do Eligible Mathematics Teachers go?," The Mathematics Teacher, 48:397-400, October, 1955.

8 H. J. Ettlinger, "Mathematics as a Profession," The Mathematics Teacher, 50:140-143, February, 1957.

9 Dr. Benigno Aldana, "Let's See to It That the School

of the root causes in the deterioration of high school instruction, according to him, is the disparity of salaries between high school teachers and elementary school teachers which leads to the exodus of qualified competent teachers to the elementary schools after complying with the necessary professional requirements. As a result, BSE teachers who are either new or inexperienced are left to teach in the high school. And, as a consequence, present salary discrimination creates a frustrating and demoralizing effect on the teachers, resulting in the deterioration of instruction.

2. Neglect on the part of the educators and architects of the mathematics program in this country. In this country, the post-war years have witnessed a decline in the teaching of mathematics. In fact, Jansen says:

> Today there is hardly a field of knowledge that has not been impregnated with mathematical modes of exposition. Yet the high school teachers in these other fields of knowledge are for the most part ignorant of mathematics and mathematical modes, and hence avoid all mention of mathematics in the subjects they teach. Many of these teachers are afraid of mathematics.[10]

The Philippine school curriculum has been so much pruned of its mathematics content that students graduating from the high school are almost mathematical illiterates. It is indeed

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System Keeps Steadfast on the Road to Excellence," In the Grade School, 11:230-233, September, 1962.

[10] Henry S. Jansen, "The Relation of Mathematics to the Core Curriculum," The Mathematics Teacher, 45:428, October, 1952.

disastrous to leave out the courses in algebra and geometry, since one has nothing worthy of the student's vigorous mental attack that can take their place.[11] The general concept--and a mistaken concept at that--that mathematics is a subject too difficult for the average student and must, therefore, be reserved for the gifted only, aggravates the situation. Both teachers and students tend to shy away from this study. In the classroom everything that alludes to mathematics or any form of quantitative evaluation is evaded.

It was Dadourian who said:

> One of the sources of troubles that some students have with mathematics is the average student's notion that the study of mathematics requires a special kind of talent, and that mathematics, beyond the elements of arithmetic, is of little or no value to him unless he is planning to become a scientist or an engineer. These two notions produce fear of the subject and apathy toward it . . . .
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> This source of trouble is created mainly by the general public, including teachers in fields other than mathematics, engineering, and the natural sciences . . . .[12]

On top of this general public aversion to mathematics is the fact that the teaching of mathematics has been sadly

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[11] David E. Smith, "Mathematics in the Training for Citizenship," Selected Topics in the Teaching of Mathematics (Third Yearbook of the National Council of Teachers of Mathematics [New York: Bureau of Publications, Teachers College, Columbia University, 1928]), pp. 11-23; as cited by Jack D. Wilson, "Trends in Geometry," The Mathematics Teacher, 46:67, February, 1953.

[12] H. M. Dadourian, "How to Make Mathematics More Attractive," The Mathematics Teacher, 53:548-551, November, 1960.

neglected during the years following World War II. Lately, however, the National Science Development Board which was created primarily for the upgrading of the teaching of science, took cognizance of the prior need for upgrading mathematics instruction. This is a healthy sign of progress, and one can only hope that the general public will adopt a responsive attitude to this program.

3. Antiquated and outdated pattern of instruction--the traditional pattern. For years one has adhered to the traditional pattern of teaching mathematics to the extent that it has become the established pattern. While algebra and geometry are defined as courses in reasoning, it is generally recognized that in practice their study has often degenerated into sterile memory work. They are no longer vehicles for teaching clear and critical thinking.[13] They have lost their values as disciplines and they no longer appeal to the intellect. More emphasis is given to the teaching of rules for solving special problems than to the teaching of young people to think mathematically.[14]

The U. S. Commission on Post-War Plans uses the term "traditional mathematics" to describe this pattern of instruction. By this they mean "the sequential courses" or what may

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[13] Jack D. Wilson, "Trends in Geometry," The Mathematics Teacher, 46:67, February, 1953.

[14] Kenneth O. May, "Which Way Precollege Mathematics?," The Mathematics Teacher, 47:303-307, May, 1954.

be called compartmentalized mathematics in which arithmetic, algebra, and geometry are taught independently of each other in the different grades and years. According to the Second Report of the Commission, this pattern of instruction in mathematics does not meet the mathematical requirements of technology and the present mode of life. Leadership in these fields presupposes an extent and range of mathematical scholarship far greater than schools have traditionally made available. A fraction of the schools' student body--unfortunately no one knows how large--will need more mathematics than educators, or even mathematics teachers, have realized.[15] The Commission also maintained that in most schools the teaching of mathematics is woefully out of date as regards both subject matter and methods.[16] The present rote-teaching of facts, drills, skills and knowledge, as a means of establishing understanding and meaning in mathematics should be a thing of the past. Facts taught in isolation with the hope that they will take their proper places in problems when they are needed is mostly a waste of time and energy; isolated facts will not transfer to problems to help pupils when they need them. Also drills

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[15] Commission on Post-War Plans, "The First Report of the Commission on Post-War Plans," The Mathematics Teacher, 37:230-231, May, 1944.

[16] Ibid., p. 232.

given before a pupil fully understands the meaning of a mathematical idea or principle, can do the pupil more harm than good.[17]

Certainly "rote" and "telling" types of teaching, which have been so prevalent in the classrooms in the past, have had much to do with making pupils in the elementary and secondary schools dislike mathematics. By using suitable functional materials and by training mathematics teachers to be experts in the functional type of teaching, it is hoped that these teachers will be able to eliminate much of this hatred and other unfavorable attitudes towards mathematics and save much time and talent.[18]

Tradition versus innovation. In the light of the foregoing, it is imperative that immediate steps be taken if a strong manpower is to be developed that will run the native industries efficiently and build an enlightened citizenry to constitute a democratic society. The experience of most teachers in the teaching of college mathematics has convinced them that, what little the high school graduates know of mathematics is very superficial. They have to some extent acquired a certain amount of skill in the manipulation of

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[17] William A. Gager, "The Functional Approach to Elementary and Secondary Mathematics," The Mathematics Teacher, 50:32, January, 1957.

[18] Ibid., p. 33.

mechanical processes, but they are definitely lacking in the power of intuitive and analytical reasoning. They have not developed logical and critical thinking, characteristic of the science of mathematics.[19]

In his retiring presidential address before the Mathematical Association of America, MacLane said:

> Education in mathematics needs the most imaginative and vigorous reforms for it is now beset by numerous troubles and inadequacies. These are internal troubles. Many of our courses cleave valiantly to a weak and obsolete tradition . . . .[20]

In this statement, MacLane has called attention to a revision problem which is growing in importance and is demanding more serious attention. This idea has found corroboration in works and proceedings of many workshops, conferences, and symposiums on mathematics education both abroad and in the Philippines. And what are these inadequacies in the teaching of mathematics that MacLane referred to? Reeve said in part:

> Traditional courses in mathematics have delayed the presentation and consideration of much interesting valuable material which may well be used to give the student, early in his career, an idea of what mathematics means, and something of the wonderful scope of its applications.[21]

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[19] B. L. Dodds, "Critical Responsibilities in Education Today," The Mathematics Teacher, 50:136-139, February, 1957.

[20] Saunders MacLane, "Of Course and Courses," The American Mathematical Monthly, 61:151-157, 1954.

[21] William D. Reeve, "General Mathematics in the Secondary School," The Mathematics Teacher, 47:78, February, 1954.

Jansen has this to say about traditional mathematics:

> Mathematics is a system of ideas and to make it seem nothing more than a group of discrete manipulative skills, unrelated and unorganized is to limit its usefulness and to distort its meaning.[22]

The same idea was presented by Professor McConnell in a penetrating discussion of "Recent Trends in Learning Theory," when he said:

> In the case of arithmetic, attempts to psychologize the subject appear to have damaged it both logically and psychologically. By decomposing it into a multitude of relatively unrelated connections or facts, psychologists have mutilated it mathematically, and, at the same time by emphasizing or encouraging discreteness and specificity rather than relatedness and generalization, they have distorted it psychologically. They have obscured the systematic character of the subject, and have created a doubtful conception of how children learn it.[23]

The Bureau of Public Schools in an educational series also concurred with this idea:

> . . . in the lower high school level it is better to present mathematics as a unified whole, stressing the natural relationships between the various traditional independent branches. Mathematics is composed of many related procedures. Not only are there dependencies within each field of mathematics, but the various fields are also interrelated.[24]

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[22] Jansen, <u>op</u>. <u>cit</u>., p. 434.

[23] J. R. McConnell, "Recent Trends in Learning Theory," (Arithmetic in General Education, <u>16th</u> Yearbook, National Council of Teachers of Mathematics, [New York: Bureau of Publications, Teachers College, Columbia University, 1941]), p.275; as cited by Henry S. Jansen, "The Relation of Mathematics to the Core Curriculum," <u>The Mathematics Teacher</u>, 45:434, October, 1952.

[24] Cortes, <u>loc</u>. <u>cit</u>.

While Professor McConnell was referring specifically to arithmetic, his statement is also applicable, and much more so, to algebra and geometry. Differentiation in the study and treatment of the component parts of the science does more damage than good. It develops into confusion and the concept of unrelatedness of the various areas of the subject. On the other hand, integration and organization around some unifying principle would establish order and relatedness. McConnell further states that "as learning proceeds, one should construct more inclusive and systematic organizations of ideas and processes governed by the fundamental structure of the number system."[25]

Surely no one would support the idea that unorganized experience is educative. Certainly the science of mathematics can never be appreciated if it is "decomposed into a multitude of relatively unrelated connections or facts." Yet, that is exactly the status of the mathematics instruction and unless this is remedied by making available the process of unification and generalization, mathematical skills can never be used successfully in other situations.[26]

<u>Reform is in the air</u>. Most forward looking mathematics teachers agree that one must stop compartmentalizing the mathematics subjects and years and proceed to fuse algebra and

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[25] McConnell, *op. cit.*, p. 273.

[26] Jansen, *op. cit.*, pp. 434-435.

geometry, that one may even include trigonometry, analytics, and calculus in a complete development of the science of number and space.[27] They have come to realize that a completely organized course in informal geometry, arithmetic, algebra, demonstrative geometry, trigonometry, analytic geometry, and calculus beginning with the first year of high school, and designed to show their various relations, would reinforce and supplement each other in useful ways. It would make the science of mathematics more interesting and meaningful. The course would develop more power, and produce better results than one has been getting from the traditional pattern. Recognizing this fact some colleges and universities have revised their traditional programs and now have unified courses. According to some proponents of this type of program, such courses contain much the same material as the traditional type, but it is presented in a manner which unifies and builds mathematics as it really is. The effectiveness of these unified courses from the general standpoint surpasses the effectiveness of the traditional courses.[28]

History of the movement for integration. The idea of integration in mathematics is not really new. A comparative

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[27] Ibid., p. 427.

[28] Lyle J. Dixon, "Mathematics and General Education," The Mathematics Teacher, 48:206, April, 1955.

study of mathematics textbooks will reveal a tendency toward correlation of subjects particularly in the collegiate level. Recent textbooks in mathematics show changes in content, organization and exposition, which are decided departures from those of pre-war books.

Professor Davis said on this point:

> The movement to make mathematics more vital and functional in the life of the student was led by Perry in England, Borel in France, and Klein in Germany, and first strongly advocated in this country by E. H. Moore in his presidential address (1902) before the American Mathematical Society. They urged the teaching of mathematics from the viewpoint of greater fusion of subject matter, functional relationships, and interpretation for understanding and transfer purposes.[29]

In his work, "Discussion on the Teaching of Mathematics," John Perry said:

> Great fields of thought are now open which were unknown to the Alexandrian philosophers. If we begin our study as the Alexandrian philosophers did, with their simplest ideas in arthmetic and geometry, we shall get stale before we know much more than they did. If we begin assuming more complex things to be true (although I do not like to assume that in truth any idea is more complex than another) as we have done in arithmetic, as we ought to do in other parts of mathematics without becoming stale, we may know of all the modern discoveries. We shall get the same intellectual training with more knowledge.[30]

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[29] David R. Davis, The Teaching of Mathematics (Reading, Massachusetts: Addison-Wesley Publishing Co., Inc., 1951), p. 100.

[30] John Perry (ed.), Discussion on the Teaching of Mathematics (New York: The Macmillan Company, 1902), p. 13; as cited by William D. Reeve, "General Mathematics in the Secondary School," The Mathematics Teacher, 47:76, February, 1954.

Outside of Perry's influence there was the impetus given the reorganization movement by the presidential address delivered by Moore,[31] of the University of Chicago, before the American Mathematical Society at its annual meeting in 1902.

During recent years there developed among progressive teachers a very significant movement to deviate from the traditional rigid compartmentalization of arithmetic, algebra, and geometry, the study of each of which is completed before another is begun, and to follow the direction of breaking down the barrier separating these subjects. The idea is to organize the subject-matter in a way that will offer a psychologically and pedagogically more effective approach to the study of mathematics.

From another report of the National Committee in Mathematical Requirements under the auspices of the Mathematical Association of America, one read as follows:

> There has thus developed the movement toward what are variously called "composite," "correlated," "unified," or "fused," "general" courses. The advocates of this new method of organization base their claims on the obvious and important interrelations between arithmetic, algebra, and geometry (mainly intuitive), which the student must group before he can gain any real insight into mathematical methods and which are inevitably obscured by a strict adherence to the conception of separate

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[31] E. H. Moore, "On the Foundation of Mathematics," *A General Survey of Progress in the Last Twenty-Five Years* (First Yearbook, National Council of Teachers of Mathematics, 1926), pp. 32-57; Reeve, *loc. cit.*

"subjects." The movement has gained considerable new impetus by the growth of the junior high school, and there can be little question that the results already achieved by those who are experimenting with the new methods of organization warrant the abandonment of the extreme "watertight compartment" method of presentation.

The newer method of organization enables the pupil to gain a broad view of the whole field of mathematics early in his high school course. In view of the very large number of students who drop out of school before graduation . . . or who for other reasons then cease their study of mathematics, this fact offers a weighty advantage over the older type of organization under which the pupils studied algebra . . . to the complete exhaustion of all contact with geometry.[32]

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[32] The National Committee on Mathematical Requirements, The Reorganization of Mathematics in Secondary Education (a Report by The National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America, Inc, 1923), pp. 12-13, and 77.

CHAPTER III

PROCEDURES

As aforementioned, this is an experimental research project designed to make a comparative study of the two ways of teaching algebra and geometry in the high school--the traditional type and the integrated type. In carrying out this experiment the writer employed the scientific method of approach to guide her in her procedure and analysis. This approach consists of the following:

Sensing a significant problem. As a mathematics teacher for eight years, the experimenter had become aware of the generally low achievements of high school graduates in mathematics and their inadequate preparation for the study of college mathematics. This was confirmed by fellow teachers. This felt situation raised the problem of upgrading the teaching of mathematics in the secondary schools, a problem also felt by mathematics teachers in many different countries, who have sought some adequate solution to it.

Defining the problem situations. During her many years of experience and observations, the writer has noted that there are a number of reasons for this situation. In Chapter I, these situations are classified into problem areas. Each of these areas can be studied independently of the other, all designed to upgrade the teaching of mathematics.

Isolating one major idea of the problem. In this thesis the investigator selected the classroom-centered area for her investigation. But this is still quite broad, and so she further limited herself to the comparative study of a new type or technique of teaching the subject, the integrated type, as against the traditional type.

Selecting the best means to employ in making the comparative study. By the nature of the investigation, it is obvious that the experimental method would best serve the purpose; so the writer adopted this method of approach in determining the relative merits of these two types of teaching the subject.

Studying contributing factors. As is true of all experimental projects, there were several factors which could influence the results. The experimenter made a careful study of all the variables which could influence the results of the investigation.

Controling the variables. After considering all the variable factors, the experimenter took the necessary steps to nullify their unwanted influence. Some of these, fortunately, were already inherently controlled by the choice of the group of students to use for study. Among these were

(1) the age factor, (2) the sex factor, (3) the study load factor, and (4) the socio-economic factor.

1. The age factor. The averages of the ages of the cases in the two classes were practically equal.

2. The sex factor. The students included in the experiment were predominantly male. There were two females in each group.

3. The study load factor. All the students involved were carrying the same study load, which is the regular curricular offering for freshmen in the College of Engineering.

4. The socio-economic status of the students. In general, the students used in the experiment belong mostly to middle-class families. Most of them were boarding in middle-class homes. There were no extremes that the experimenter was aware of.

For the purpose of the experiment, therefore, the above-enumerated variables were rendered constant enough to permit the interpretation of the results of the experiment to be in terms of the effect of the teaching approaches used.

There were, however, other variable factors which needed special attention in order to equate the two groups. These were (1) the general intelligence and abilities of the students involved in the experiment, (2) the teacher, (3) the courses offered, and (4) the environmental conditions during class recitations.

Organization of the classes. For the purpose of this experiment, two classes each consisting of thirty-five students were constituted at the beginning of the term when the study was conducted. For convenience the experiment was conducted in a Mathematics 110 (Review of High School Mathematics and Solid Geometry) class, with students in the first year of the College of Engineering at Central Philippine University as cases. Among the reasons for the selection of the subjects for experimentation were:

1. The students had practically the same background in Arithmetic, Algebra, and Geometry. They were all new high school graduates who had complied with the minimum mathematics requirement for admission to the College of Engineering.

2. Under the circumstances, the experiment could not be conducted in the first year of high school without disrupting the high school program, since the curricular mathematics offering in the high school is of the compartmentalized type. The experiment would have been in violation of the Bureau of Private Schools' prescribed curriculum. On the other hand, the subject matter of Mathematics 110 in the first year, College of Engineering exactly fits the purpose of the experiment.

In order to further make the classes comparable, a survey test (Appendix A)[1] was first given, and the results

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[1] *Infra*, pp. 84-91.

were used as the basis for constituting the two classes. As a result of this testing, the students were divided into two parallel groups. The mean scores were almost equal for the two groups. The values of the means of the 1962-1963 survey test, as shown in Tables I and II, were 53.88 and 53.57 for the experimental group and the control group, respectively; that of the 1963-1964 survey test, as shown in Tables III and IV, were 49.34 and 49.68 for the experimental group and the control group, respectively.

The students' high school ratings in mathematics were disregarded since they came from many different high schools with different grading standards. From previous experience it had been learned that the ratings appearing in Form 137 were not very reliable criteria for determining the relative abilities of the students on any subject. On the contrary, the entrance test ranged the students on the basis of a common factor. Thus the student factor was controlled for the purpose of the experiment. After the final organization, the two classes were identified as the experimental group and the control group, respectively.

The teaching of the class and the teacher factor. The two classes were taught by the same teacher, the experimenter herself. The decision to do so was dictated by the desire to eliminate the problem of looking for two teachers who are truly comparable. The experimenter, however, was very well

TABLE I

CALCULATION OF THE MEAN (M) FROM RAW SCORES
OF THE 1962-1963 SURVEY TEST
EXPERIMENTAL GROUP

Raw scores (X)	Frequency (f)	fX	Calculations*
93	1	93	
86	1	86	
84	1	84	
79	1	79	
78	1	78	
71	1	71	
70	1	70	
67	1	67	
64	1	64	
62	2	124	
59	1	59	
58	1	58	
57	1	57	
56	1	56	
54	1	54	
52	1	52	
49	2	98	
48	2	96	
47	1	47	
45	2	90	
43	1	43	
42	1	42	
40	2	80	
39	1	39	
38	1	38	
36	1	36	
35	1	35	
34	1	34	
31	1	31	
25	1	25	
Total	35	1886	

Calculations*:

$$M = \frac{E(fX)}{N}$$

$$= \frac{1886}{35}$$

$$= 53.88$$

*M = the arithmetic mean

E = the sum of the quantity that follows (here, all the fX's)

X = raw scores

fX = a raw score multiplied by its corresponding frequency (f)

N = total number of cases, equal to the sum of the frequencies (Ef)

TABLE II

CALCULATION OF THE MEAN (M) FROM RAW SCORES
OF THE 1962-1963 SURVEY TEST
CONTROL GROUP

Raw scores (X)	Frequency (f)	fX	Calculations*
90	1	90	$M = \frac{E(fX)}{N}$
88	1	88	
82	1	82	
80	1	80	$= \frac{1885}{35}$
74	1	74	
73	2	146	
66	3	198	$= 53.57$
65	1	65	
60	2	120	
59	1	59	
57	1	57	
52	3	156	*M = the arithmetic mean
50	2	100	
48	1	48	E = the sum of the quantity that follows (here, all the fX's)
47	1	47	
45	1	45	
44	1	44	
43	1	43	X = raw scores
41	1	41	
38	1	38	fX = a raw score multiplied by its corresponding frequency (f)
37	3	111	
36	1	36	
33	1	33	
31	1	31	N - total number of cases, equal to the sum of the frequencies (Ef)
30	1	30	
23	1	23	
Total	35	1885	

TABLE III

CALCULATION OF THE MEAN (M) FROM RAW SCORES
OF THE 1963-1964 SURVEY TEST
EXPERIMENTAL GROUP

Raw scores (X)	Frequency (f)	fX	Calculations*
89	1	89	$M = \frac{E(fX)}{N}$
81	1	81	
77	1	77	
73	1	73	$= \frac{1727}{35}$
69	1	69	
67	1	67	
64	1	64	$= 49.34$
62	1	62	
57	1	57	
56	1	56	
55	2	110	
53	1	53	*M = the arithmetic mean
50	2	100	
48	3	144	E = the sum of the quantity
46	2	92	that follows (here, all
45	1	45	the fX's)
44	1	44	
43	1	43	X = raw scores
42	2	84	
40	1	40	fX = a raw score multiplied
38	1	38	by its corresponding
35	1	35	frequency (f)
34	2	68	N = total number of cases,
30	1	30	equal to the sum of the
28	1	28	frequencies (Ef)
27	1	27	
26	1	26	
25	1	25	
Total	35	1727	

TABLE IV

CALCULATION OF THE MEAN (M) FROM RAW SCORES OF THE 1963-1964 SURVEY TEST CONTROL GROUP

Raw scores (X)	Frequency (f)	fX
87	1	87
82	1	82
76	1	76
73	1	73
71	1	71
70	1	70
64	1	64
63	1	63
62	1	62
57	2	114
56	1	56
55	1	55
50	1	50
49	2	98
48	2	96
45	2	90
44	1	44
43	1	43
42	1	42
41	2	82
39	2	78
35	2	70
34	2	68
30	1	30
28	1	28
27	1	27
20	1	20
Total	35	1739

Calculations*

$$M = \frac{E(fX)}{N}$$

$$= \frac{1739}{35}$$

$$= 49.68$$

*M = the arithmetic mean

E = the sum of the quantity that follows (here, all the fX's)

X = raw scores

fX = a raw score multiplied by its corresponding frequency (f)

N = total number of cases, equal to the sum of the frequencies (Ef) aware of the danger of a bias in the teaching when the teacher favors one of the factors to be tested. At the very outset she made it a point to adopt the scientific attitude. She did her best to equalize everything about her teaching in both classes except for the patterns used. There was, however, one point where the experimental group was a little at a disadvantage. The teacher had had years of experience in teaching after the traditional pattern while she taught with the new pattern for the first time and without the use of established material. It was for this very reason that she repeated the experiment during the following year. In teaching the experimental classes, the teacher had the benefit of being guided by the head of the department of mathematics who helped her along in the methods and techniques of teaching. There can be no doubt that she must have made some improvement during the second experimentation and removed the handicap of uneven teacher preparation from the experiment.

<u>The coverage factor</u>. The two groups covered the same area prescribed by the Bureau of Private Schools.

<u>The environmental factor</u>. Since these experiments were conducted in the same college (College of Engineering) and in the same school (Central Philippine University), this variable was fairly well controlled. Precautions were even taken to assign the two classes each semester in the same room.

The time of the day for the classes was also considered. During the first experiment the control group met from 5:30 to 6:30 P.M. while the experimental group met from 6:30 to 7:30 P.M. During the second experiment the time order was reversed; the control group met from 5:30 to 6:30 P.M. while the experimental group met from 3:30 to 4:30 P.M.

The pattern and technique of teaching. This was the only variable factor that was left unequated, since this was the area for study in the experiment. With the first group the traditional pattern was used, as it had been used since the Mathematics 110 course was first introduced in the College of Engineering in 1955, and as it is used in the high school at present. The nature of this pattern is that arithmetic, algebra, and geometry, are each taught separately, that is, the teaching is compartmentalized both in point of time and subject matter. When solving problems in algebra, the use of geometry was excluded and, vice-versa, except in certain established practices. Explanations and discussions of principles in algebra were done without reference to geometry, and explanations in geometry were conducted without mention of algebra. One was not resorted to in order to clarify the other. Except in rare cases where it has become a common practice to use diagrams and figures in order to understand and interpret problems in algebra, care was taken that each subject did not use aids from the other. The teaching of these subjects became very highly compartmentalized. This was definitely the pattern used in the control group. The experimenter would like to make it understood that she had not made a special effort to accomplish absolute compartmentalization. She only adhered to the regular pattern that she had been following for many years while teaching the separate subjects both in the high school and in college.

In the experimental group the experimenter tried to integrate algebra with the teaching of geometry and geometry with the teaching of algebra. Her difficulty here lay in the absence of a regular textbook for this particular method. Her sources of materials for this purpose were the following: (1) a few of the latest books on modern mathematics, (2) articles in current mathematics journals, and (3) the assistance and suggestions of the Head of the Departments of Mathematics and Physics of Central Philippine University.

In teaching both groups the same general outline was used. This care had to be taken in order to ensure the same coverage. The difference lay in the approaches to the teaching of the different items in the outline. In the experimental group the teaching was modified to allow integration whenever and wherever possible. The author must admit here that it would have been quite difficult to integrate the two subjects completely. According to Professor Fehr,[2] in a

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[2] Howard F. Fehr, "Using Algebra in Teaching Geometry," The Mathematics Teacher, 45:561, December, 1952.

paper presented at the Thirtieth Annual Meeting of the National Council of Teachers of Mathematics, this would necessitate the establishment of a new set of postulates in order to effect a total integration. But there is no such set of postulates existing at the present time. And so, wherever the experimenter could not see any way of integrating the subjects, she taught the particular item in the outline in the same manner in both groups.

The experimental procedure, therefore, consisted mainly in the exposition of how algebra and geometry may be employed together or simultaneously in the teaching of a mathematical concept or principle. In other words, when dealing with a purely abstract algebraic principle, geometry was used to illustrate the concept, and when teaching a geometric concept, algebra was brought in to show how generalizations are abstracted from specific problems. As was said, this was not possible in all cases. Sometimes the experimenter had to dispense with integration.

But by way of illustration of the new pattern of instruction, the following are samples picked up at random. These are just brief summaries of how integration was done in a few items. Many of these are found in textbooks and current mathematics journals, but in isolated forms, and without any effort at correlation.

Examples of integration. The following are some of the examples of integration:

1. The teaching of signed numbers. To beginners in elementary algebra, signed numbers mean nothing more than numbers preceded by plus (+) or minus (-) signs. This is practically what most high school graduates would give as definition. Such a definition would be like calling the picture of a man, or the written name of a man, the man himself. Some teachers would correct this misconception by telling the class that numbers preceded by the plus sign (positive) are numbers greater than zero (0) and numbers preceded by the minus sign (negative) are less than zero (0). Some may say that they are directed numbers. Both of these so-called corrections or further explanation, though correct, would still be very vague, and would leave the students still wondering.

But if the definition or explanation is clarified by the use of directed lines (Figure 1), then the students will begin to attach some significance to positive (+) and negative (-) numbers by looking at the picture. This would be just the same as showing the picture of a man instead of trying to make a verbal or written description of his face.

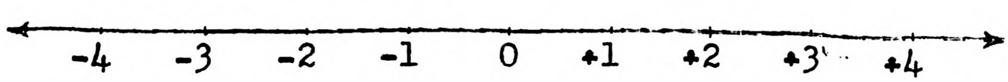

FIGURE 1

DIRECTED LINES

2. The process of multiplication. Multiplication in arithmetic can be translated into proportion in algebra, and properties of similar triangles in geometry.

Given: Line a = ______________

Line b = __________

Required: To multiply two lines, that is, to find a line which is the product of the lines a and b.

Procedure: Draw two lines M and N meeting at O (Figure 2), and measure a unit length OX. Measure OY = a and XZ = b. Draw XY and through Z draw a line parallel to XY meeting ON at W. YW is equal to ab, the product of lines a and b.

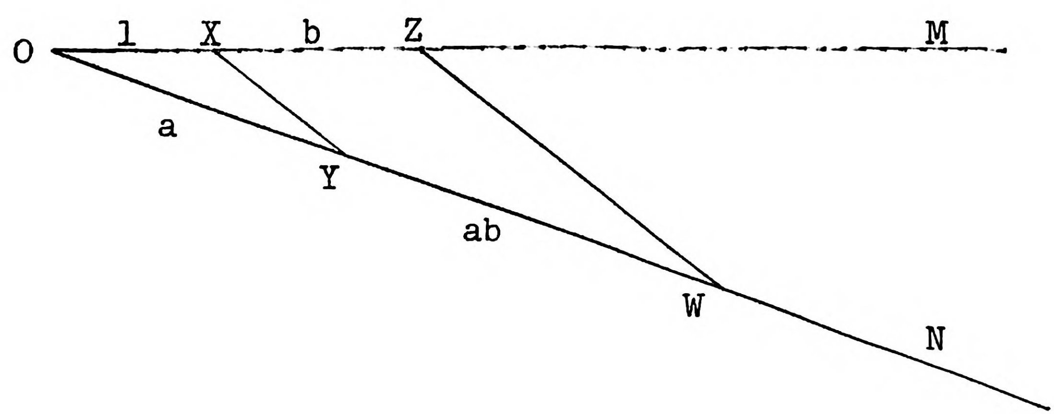

FIGURE 2

GEOMETRICAL REPRESENTATION OF MULTIPLICATION

Proof:

1:a = b:YW	A line parallel to one side of a triangle divides the other sides into segments that are proportional.
YW = ab	In any proportion, the product of the means is equal to the product of the extremes.

3. Finding the area of a rectangle. The elementary concept for any rectangle is that, area is the product of the measure number of the length and the measure number of the width. This is applied in arithmetic on rectangles of different dimensions, thus: for a rectangle of length 4 units and width 3 units (Figure 3), the area is 12 square units; for a rectangle of length 6 units and width 4 units, the area is 24 square units; and for a rectangle of length 5 units and width 2 units, the area is 10 square units, and so forth.

In each of the above examples, the dimensions are specific and so are the corresponding areas. (This is arithmetic).

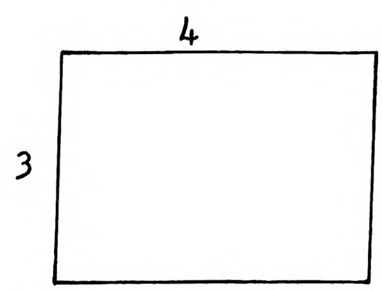

FIGURE 3

ARITHMETICAL REPRESENTATION OF THE AREA OF A RECTANGLE

The above specific cases can be generalized and expressed in a single pattern, by designating a symbol to represent the length of any rectangle and another symbol to represent the width of the same rectangle. Thus, in Figures 4a, 4b, and 4c, if L and W are designated to represent the length and width respectively, of any rectangle, then area equals length times width (A = LW). (This is algebra or "generalized" arithmetic).

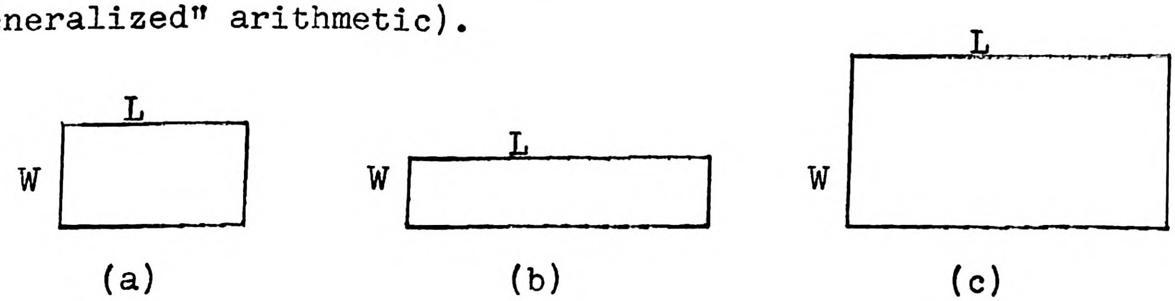

FIGURE 4

ALGEBRAIC REPRESENTATION OF THE AREA OF A RECTANGLE

In geometry, one defines area as the number of unit squares that the rectangle contains (Figures 5a and 5b).

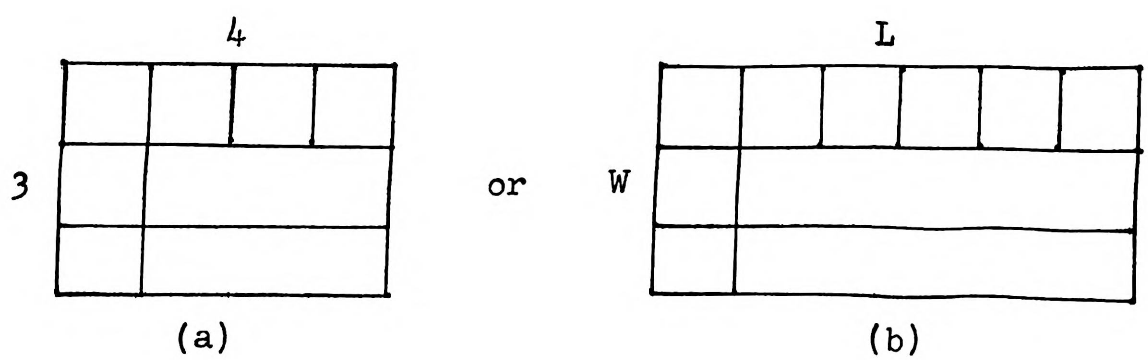

FIGURE 5

GEOMETRICAL REPRESENTATION OF THE AREA OF A RECTANGLE

From the above figures, it is shown that there are 3 (or W) rows of 4 (or L) squares each, thus making the area 4 x 3 squares = 12 squares or L x W squares = LW squares.

4. Illustrating the square of a binomial. The general pattern of teaching the square of a binomial in algebra is to give a few examples such as $(a + b)^2$, $(x + y)^2$, and so forth. Then after performing the regular process of long multiplication the answers are studied for the relations of the terms in the product to the terms of the binomial. On the basis of this observation a rule is established in the form of an empirical pattern for the process, thus, $(x + y)^2 = x^2 + 2xy + y^2$. A geometrical demonstration of this principle is given in Figure 6.

Let *x* be the side of the original square. Then the area is x^2. If the sides are increased by *y* to make them $(x + y)$, then the total area is $x^2 + xy + xy + y^2$ or $x^2 + 2xy + y^2$.

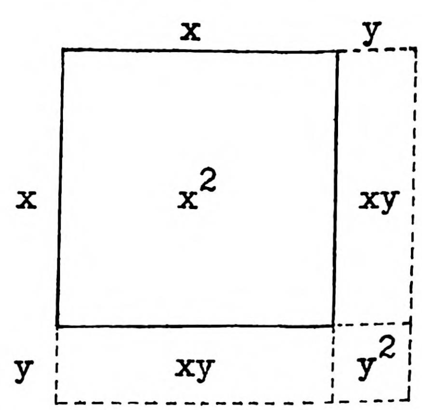

FIGURE 6

GEOMETRICAL REPRESENTATION OF THE SQUARE OF A BINOMIAL

5. <u>The process of evolution</u>. Any high school graduate should have no difficulty in extracting the square root of any positive number. But when students are asked to explain why they multiply the root already found by 20 to get the trial divisor, their only answer is, "Our high school teacher in mathematics gave that to us." In the traditional pattern of teaching this operation, students are arbitrarily told no more than that they should multiply the root already found by 20 to get the next trial divisor. The teacher concentrates on the proper manipulation of the process.

The only way to explain this step in the process is by referring back to the binomial expansion in algebra, that is, $(a + b)^2 = a^2 + 2ab + b^2$, in which "a" stands for the 10's digit and "b" stands for the units digit. After obtaining the first digit of the root, $\underline{2ab + b^2}$ remains. In order to obtain the next digit this remainder is first factored into $b(2a + b)$. Since "a" stands for the 10's digit, the trial divisor should be $(2 \times 10a)$ instead of $(20 \times a)$ as is the common practice. Here is where algebra explains an arithmetical process.

The writer has not yet met a high school graduate who can extract the cube root of a number. But referring again to the "binomial theorem," this can be reduced to the pattern: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. After the first digit of the root is taken, the remainder is $3a^2b + 3ab^2 + b^3$.

Factoring this remainder into $b(3a^2 + 3ab + b^2)$, the quantity in parenthesis is obtained as the complete divisor. The value $3a^2$ is used for obtaining the trial divisor, but since "a" stands for the 10's digit, therefore the trial divisor becomes $3(10a)^2$. By this method, the students begin to learn the process of evolution in both arithmetic and algebra.

Geometry can also be used in extracting the square root of a given line.

Given: Line a = ______________

Required: To extract the square root of the given line a.

Procedure: On the line OC (Figure 7), measure OA equal to one unit length. Measure AB equal to the given line a. Draw AD perpendicular to OC. With OB as diameter, draw a semicircle intersecting AD at E. AE is the square root of the given line a.

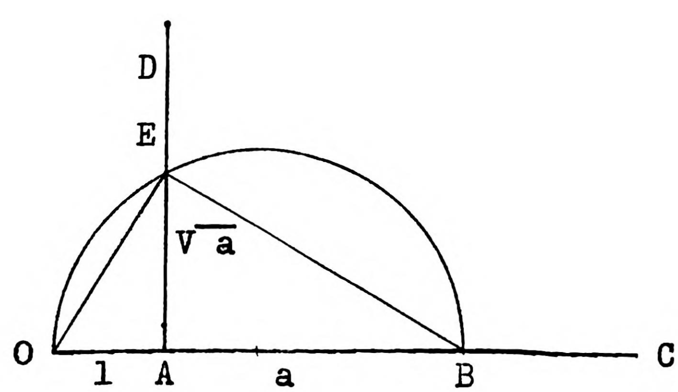

FIGURE 7

GEOMETRICAL REPRESENTATION OF EXTRACTING OF SQUARE ROOT

Proof:

$1:AE = AE:a$	In a right triangle, the altitude upon the hypotenuse is the mean proportional between the segments of the hypotenuse.
$AE = \sqrt{a}$	In any proportion, the product of the means is equal to the product of the extremes.

6. The process of involution. Another demonstration on how an arithmetic and/or algebraic process can be interpreted geometrically is the fundamental operation of involution (raising to a power).

Involution is simply continued multiplication by the same number, that is, $2^5 = 2 \times 2 \times 2 \times 2 \times 2$, and $(a + b)^4 = (a + b)(a + b)(a + b)(a + b)$, and can be performed easily in both arithmetic and algebra. Sornito[3] has demonstrated this process geometrically using two different methods, both based upon the simple properties of right triangles, as shown in the following (Figures 8 and 9):

Given: Line a = ____________

Required: To find the nth power of a given line a.

Procedure: (First method)

Draw two lines perpendicular to each other as shown in Figure 8. Measure AO = 1 unit length.

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[3] Juan E. Sornito, "Involution Operated Geometrically," The Mathematics Teacher, 48:243-244, April, 1955.

Measure OB = a. Construct a perpendicular bisector to AB, meeting XX' at x. With x as center, describe a circle passing through A and B and intersecting XX' at C. OC = a^2. Construct again a perpendicular bisector to the line BC, intersecting YY' at y. With y as center, describe a circle passing through B and C and intersecting YY' at D. OD = a^3, and so forth.

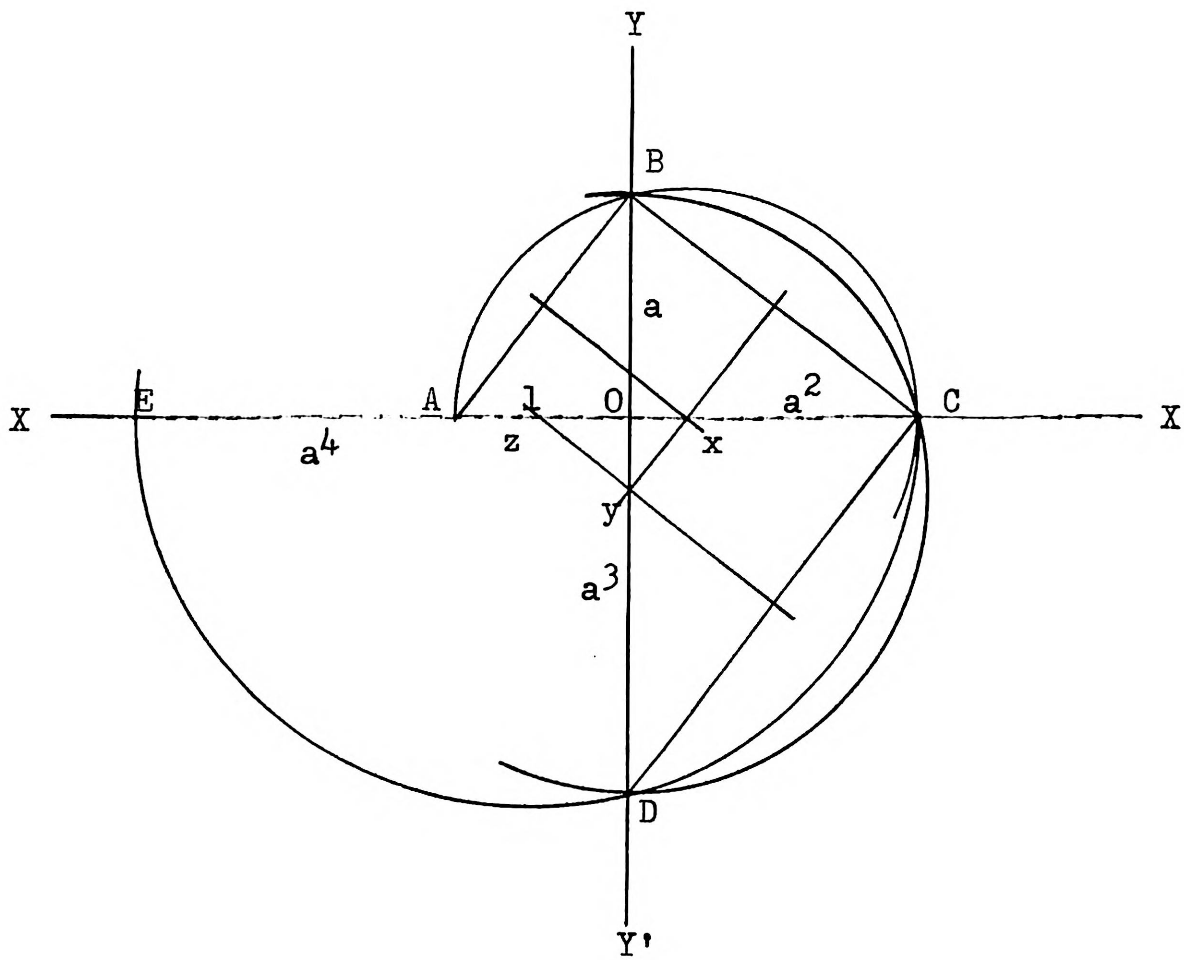

FIGURE 8

GEOMETRICAL REPRESENTATION OF INVOLUTION (FIRST METHOD)

Proof:

1. $1:a = a:OC$ — Proportions (1) and (2) use the same proposition, thus:

   $OC = a^2$

2. $a:a^2 = a^2:OD$ — In a right triangle, the altitude upon the hypotenuse is the mean proportional between the segments of the hypotenuse.

   $OD = a^3$

Procedure: (Second Method)

Let OA on the line OM equal the unit length. At A erect a perpendicular to OM. By the use of a compass measure OB equal to a, the given line. Through B draw ON. Construct BC perpendicular to ON; CD perpendicular to OM; DE perpendicular to ON; EF perpendicular to OM; and so forth. It can be proved that $OC = a^2$, $OD = a^3$, $OE = a^4$, and so forth, to a^n.

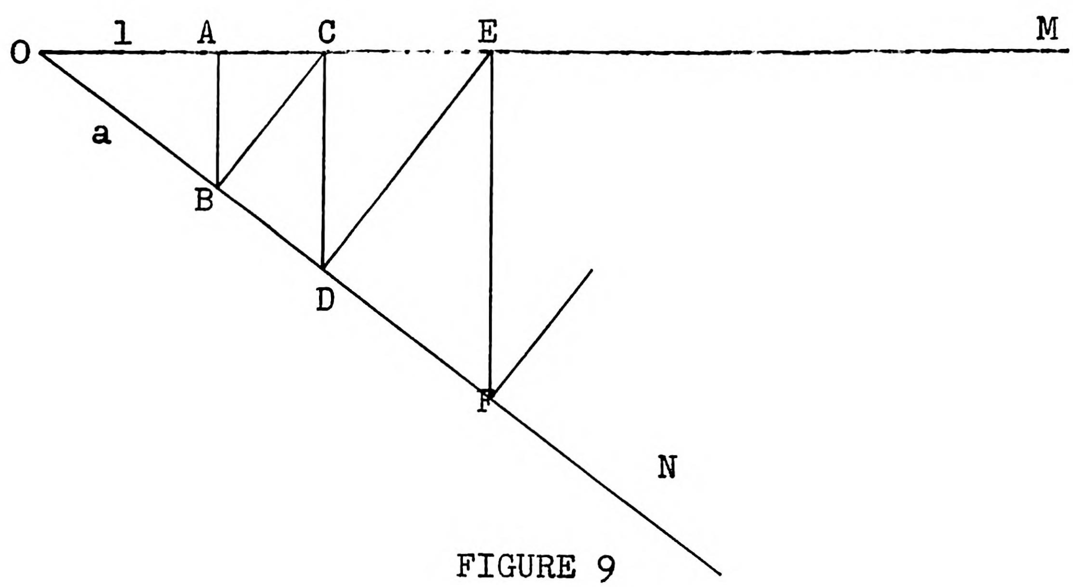

FIGURE 9

GEOMETRICAL REPRESENTATION OF INVOLUTION (SECOND METHOD)

Proof:

1. $1:a = a:OC$

   $OC = a^2$

2. $a:a^2 = a^2:OD$

   $OD = a^3$

3. $a^2:a^3 = a^3:OE$

   $OE = a^4$

Proportions (1), (2), and (3) use the same proposition, thus:

Either leg of a right triangle is the mean proportional between the hypotenuse and the projection of that leg upon the hypotenuse.

Final testing at the end of the term. Upon the termination of the term, a final test (Appendix B)[4] was administered to the two groups for the purpose of determining their relative achievements. The same sets of questions were used consisting of two types; namely, (1) exercises to test their accuracy and skill in the mechanical operations, and (2) problems to test their power to interpret, that is, their ability to translate ideas into the language of mathematics, and their ability to make complete solutions.

The papers were carefully graded, and from the results the achievements of the students in the two groups were compared. These results were carefully tabulated and statistically analyzed.

Evaluating the results of the final test. Since one is continually trying to answer questions, especially in scientific

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[4] *Infra*, pp. 92-96.

work, one may wish to answer such a specific question as this: "Is the integrated teaching of algebra and geometry superior to the traditional pattern presently followed in the secondary schools?"

It is clear that questions of this sort cannot be answered with absolute certainty, simply because any measurements of a sociological nature do not lend themselves to precise quantification. Despite this fact, one can certainly obtain a basis for judgment from the experimental results. To obtain some indication of the comparative merit of these two, the <u>null hypothesis</u> was used. One assumed that there was no difference between the traditional and the integrated pattern of teaching and that whatever differences were observed were obtained by chance.

If an estimate of the standard deviation of such a hypothetical distribution of differences is made, then it could be determined with some confidence whether or not the two samples M's came from a distribution of differences in which M is zero. If the difference between M's can indeed be shown to be a likely result of sampling error, then the <u>null hypothesis</u> cannot be rejected and it can be asserted that the differences in teaching approach had no appreciable influence on the class responses. If M's can be shown to be significantly different, one can conclude that the methods tested very probably produced significant differences in responses.

The arithmetic mean (M) is one of the measures of central tendency commonly called the arithmetic average. It is the point on the scale above which and below which the deviations are equal. The arithmetic mean or simply "the mean" for short is the most valuable measure of central tendency because it is the most stable and the most consistent. The mean is equal to the sum of the raw scores divided by their number; therefore, each score has equal weight in its determination.

The standard deviation (sigma) is defined as the square root of the mean of the squares of the deviations of the scores from their mean. It is a more stable measure of dispersion because it is not greatly affected by fluctuations in the distribution of scores. As a measure of variability, the standard deviation was used in this study because it allows the standard errors of the mean, the standard errors of the difference, and the t-ratios to be computed subsequently.

The standard error of the mean is symbolized σ_M, and literally is an estimate of the standard deviation of a distribution of randomly drawn sample M's. In other words, it is the variability of the sampling distribution of M's. The standard error of the mean is equal to the standard deviation of the raw scores of the sample divided by the square root of one less than the number of cases in the sample.

The standard error of a difference (σ_{diff}) is equal to the square root of the sum of the squares of the standard error of the mean of the experimental group and the standard error of the mean of the control group.

The t-ratio (t) is found by dividing the difference between the means of the experimental and the control group by the standard error of a difference. To determine from the table whether the value of t is significant or not at a certain level of confidence, one enters the table at the degree of freedom allowed by the experimental design. In this experiment, the degrees of freedom (D.F.) is equal to two less than the sum of the cases for both the experimental and the control group.

The steps in the analysis, therefore, are the following: the calculation of the mean (M), the standard deviation (σ), and the standard error of the mean (σ_M) from raw scores. If an estimate of the standard deviation of the sampling distribution of differences could be made, then the null hypothesis for a pair of sample M's could be tested by the formula for the standard error of a difference. And, finally, the t-ratio (t) is computed to determine the significance at a certain level of confidence.

CHAPTER IV

OBSERVATIONS, FINDINGS, AND ANALYSIS

The evaluation of the comparative merits of the two teaching approaches was based primarily upon (1) the observations made during the course of the experiment for each semester and (2) the statistical analysis of the results of the final test in each semester.

A more convincing result could have been obtained by following up the students in their higher mathematics course. This procedure, however, was beyond the control of the experimenter because another instructor took over the teaching of higher courses. The researcher, therefore, had to be contented with only one term of observation and experimentation as the basis for her conclusion. However, the fact that the experiment was conducted twice gave sufficient validity to the findings.

I. OBSERVATIONS AND FINDINGS BASED UPON THE DAILY CLASSROOM ACTIVITIES

These observations were classified under two headings as follows:

Responses of the students. At the beginning of the term the integrated pattern of teaching mathematics was explained to the students in the experimental group. They were told that in some subject matter in algebra, geometry was going to be used simultaneously to help them understand the theory or principle under consideration, and vice-versa. The experimenter could not be sure about the reaction of the students to this prior notice. It may have frightened them or it may have aroused their curiosity.

During the first two or three weeks, the students in the experimental group appeared to be a little bewildered, and their responses were very weak and slow. They seemed on their guard and reluctant to adopt the new technique. They could not easily follow the switching from algebra to geometry and from geometry to algebra, in the course of the explanation. In the course of a few weeks, however, they began to adapt themselves to the new technique, and by the middle of the term their responses were spontaneous. They began to develop the ability to "picture" abstract algebraic concepts in terms of the more perceptible language of geometry. Near the end of the term, many were already capable of demonstrating by the use of geometry simple algebraic principles, and vice-versa. They have evidently acquired some degree of thinking simultaneously in terms of both. They began to realize the "relatedness" and correlation between the two basic foundations of mathematics. In short, they have learned to develop a broader perspective of the science of mathematics.

Interpretation of problems. Another important observation made relative to the classroom activities of the experimental group was the progress they made in their ability to interpret problems. The periodic test given during the semester (a test on the interpretation of word problems), indicated the more rapid improvement in the experimental group. They have developed readiness in the use of diagrams as aid in the interpretation of problems and in expressing relationships between variables in the language of mathematics.

II. OBSERVATIONS AND FINDINGS BASED UPON THE RESULTS OF THE FINAL TEST

In an experimental research, in which an answer to a certain question is sought, mere observations made during the course of the experiment are not sufficient to establish a valid conclusion.

Observations on the class activities may leave plenty of room for doubt in that they do not lend themselves to exact measurements. They may be influenced by environmental conditions of the observer. A more valid basis for judgment, therefore, was the mathematical analysis and interpretation of the numerical and quantitative records taken. These observations were checked against the more objective analysis of the results of the final test.

Very frequently, it is desirable to test the means of two samples to determine whether there is any significant differences between them, or whether the difference, if any, is merely due to chance. The investigator in her desire to determine whether the integrated teaching of algebra and geometry is better than the present traditional pattern presently followed in the secondary schools worked with seventy subjects, thirty-five in a control group and thirty-five in an experimental group, twice; the first during the first semester of 1962-1963 and the second during the first semester of 1963-1964. The effectivity of the integrated pattern over the traditional pattern was measured by the results of the final test given at the end of the term. It was sought to determine whether the measured results for the control group differed significantly from those of the experimental group.

The question the experiment sought to answer is whether integrated teaching is more effective than the traditional method of teaching. The implied null hypothesis is that there is no difference between the integrated teaching and the traditional pattern of teaching and that any difference obtained between the mean scores in the experiments is a matter of chance. In examining the design of the experiment, thirty-five such differences (one from each subject) were made available, providing information on the experiment question.

Summary of the results of the 1962-1963 final test. From the raw scores of the experimental group (Table V), the

TABLE V

CALCULATIONS OF THE MEAN (M), STANDARD DEVIATION (σ), AND STANDARD ERROR OF THE MEAN (σ_M), FROM RAW SCORES OF THE 1962-1963 FINAL TEST EXPERIMENTAL GROUP

Raw scores (X)	Frequency (f)	fX	fX^2
148	1	148	21904
140	1	140	19600
136	1	136	18496
130	1	130	16900
122	1	122	14884
121	1	121	14641
120	1	120	14400
112	1	112	12544
109	1	109	11881
107	2	214	22898
106	1	106	11236
100	1	100	10000
96	2	192	18432
95	1	95	9025
94	1	94	8836
92	1	92	8464
88	1	88	7744
87	1	87	7569
85	2	170	14450
84	1	84	7056
80	1	80	6400
78	1	78	6084
77	2	154	11858
75	1	75	5625
70	1	70	4900
65	1	65	4225
64	1	64	4096
62	1	62	3844
58	1	58	3364
51	1	51	2601
40	1	40	1600
Total	35	3257	325557

COMPUTATION OF THE MEAN (M):

$$M = \frac{E(fX)}{N} = \frac{3257}{35} = 93.06$$

COMPUTATION OF THE STANDARD DEVIATION (σ):

$$\sigma = \sqrt{\frac{E(fX^2)}{N} - M^2} = \sqrt{\frac{325557}{35} - (93.06)^2}$$

$$= \sqrt{9301.33 - 8660.16} = \sqrt{641.17}$$

$$= 25.32$$

COMPUTATION OF THE STANDARD ERROR OF THE MEAN (σ_M):

$$\sigma_M = \frac{\sigma}{\sqrt{N - 1}} = \frac{25.32}{\sqrt{34}} = \frac{25.32}{5.83}$$

$$= 4.34$$

arithmetic mean was found to be 93.06; that of the control group (Table VI) was 80.57. The values of the standard deviation and the standard error of the mean for the experimental group were computed to be 25.32 and 4.34, respectively; that of the control group were 23.63 and 4.05, respectively.

Table VII shows the calculation of the *t*-ratio. In solving for the *t*-ratio, the difference between the means of the experimental and the control group was computed and found to be 12.49. Also the standard error of difference was computed to be 5.92, thus giving the value of 2.11 to the *t*-ratio. For 68 degrees of freedom, the *t*-ratio of 2.11 was significant at 5 per cent level of confidence, as shown in Table C of O. L. Lacey.[1]

Summary of the results of the 1963-1964 final test. Calculation of the mean from the raw scores of the experimental group (Table VIII) was found to be 90.11; that of the control group (Table IX) was 77.34. The values of the standard deviation and the standard error of the mean for the experimental were found to be 24.08 and 4.13, respectively; those of the control group were 23.05 and 3.95, respectively.

Table X indicates the computations of the standard error

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[1] O. L. Lacey, *Statistical Methods in Experimentation: An Introduction* (New York: The Macmillan Company, 1953; *infra*, p. 100.

TABLE VI

CALCULATIONS OF THE MEAN (M), STANDARD DEVIATION (σ), AND STANDARD ERROR OF THE MEAN (σ_M), FROM RAW SCORES OF THE 1962-1963 FINAL TEST CONTROL GROUP

Raw scores (X)	Frequency (f)	fX	fX^2
137	1	137	18769
128	1	128	16384
120	1	120	14400
116	1	116	13456
107	1	107	11449
104	1	104	10816
102	1	102	10404
98	1	98	9604
93	1	93	8649
92	1	92	8464
91	1	91	8281
90	1	90	8100
89	1	89	7921
87	1	87	7569
84	1	84	7056
82	1	82	6724
79	2	158	12482
78	1	78	6084
72	3	216	15552
70	1	70	4900
68	2	136	9248
67	1	67	4489
64	1	64	4096
60	2	120	7200
57	2	114	6498
53	1	53	2809
47	1	47	2209
43	1	43	1849
36	1	36	1296
Total	35	2822	246758

COMPUTATION OF THE MEAN (M):

$$M = \frac{E(fX)}{N} = \frac{2822}{35} = 80.57$$

COMPUTATION OF THE STANDARD DEVIATION (σ):

$$\sigma = \sqrt{\frac{E(fX^2)}{N} - M^2} = \sqrt{\frac{246758}{35} - (80.57)^2}$$

$$= \sqrt{7050.23 - 6491.52} = \sqrt{558.71}$$

$$= 23.63$$

COMPUTATION OF THE STANDARD ERROR OF THE MEAN (σ_M):

$$\sigma_M = \frac{\sigma}{\sqrt{N - 1}} = \frac{23.65}{\sqrt{34}} = \frac{23.63}{5.83}$$

$$= 4.05$$

TABLE VII

CALCULATION OF THE *t*-RATIO
1962-1963 FINAL TEST

Test group	Number of cases (N)	Mean (M)	Standard deviation (σ)	Standard error of the mean (σ_M)
Experimental	35	93.06	25.32	4.34
Control	35	80.57	23.63	4.05

COMPUTATION OF THE STANDARD ERROR OF A DIFFERENCE (σ_{diff}):

$$\sigma_{diff} = \sqrt{\sigma_{M_E}^2 + \sigma_{M_C}^2}$$

$$= \sqrt{(4.34)^2 + (4.05)^2}$$

$$= \sqrt{18.8356 + 16.4025}$$

$$= \sqrt{35.1318}$$

$$= 5.92$$

COMPUTATIONS OF THE DEGREES OF FREEDOM AND THE *t*-RATIO:

$$D.F. = N_E + N_C - 2 = 35 + 35 - 2 = 68$$

$$t = \frac{M_E - M_C}{\sigma_{diff}} = \frac{93.06 - 80.57}{5.92} = \frac{12.49}{5.92}$$

$$= 2.11, \text{ significant for 68 D.F.}$$

TABLE VIII

CALCULATIONS OF THE MEAN (M), STANDARD DEVIATION (σ), AND STANDARD ERROR OF THE MEAN (σ_M), FROM RAW SCORES OF THE 1963-1964 FINAL TEST EXPERIMENTAL GROUP

Raw scores (X)	Frequency (f)	fX	fX^2
144	1	144	20736
133	1	133	17689
130	1	130	16900
122	1	122	14884
118	1	118	13924
115	1	115	13225
108	1	108	11664
107	1	107	11449
106	1	106	11236
105	1	105	11025
102	1	102	10404
98	1	98	9604
97	2	194	18818
96	2	192	18432
93	1	93	8649
90	1	90	8100
88	1	88	7744
85	1	85	7225
83	2	166	13778
81	1	81	6561
80	1	80	6400
79	2	158	12482
75	1	75	5625
72	1	72	5184
69	1	69	4761
68	1	68	4624
62	1	62	3844
58	1	58	3364
54	1	54	2916
46	1	46	2116
35	1	35	1225
Total	35	3154	304488

COMPUTATION OF THE MEAN (M):

$$M = \frac{E(fX)}{N} = \frac{3154}{35} = 90.11$$

COMPUTATION OF THE STANDARD DEVIATION (σ):

$$\sigma = \sqrt{\frac{E(fX^2)}{N} - M^2} = \sqrt{\frac{304488}{35} - (90.11)^2}$$

$$= \sqrt{8699.66 - 8119.81} = \sqrt{579.85}$$

$$= 24.08$$

COMPUTATION OF THE STANDARD ERROR OF THE MEAN (σ_M):

$$\sigma_M = \frac{\sigma}{\sqrt{N - 1}} = \frac{24.08}{\sqrt{34}} = \frac{24.08}{5.83}$$

$$= 4.13$$

TABLE IX

CALCULATIONS OF THE MEAN (M), STANDARD DEVIATION (σ), AND STANDARD ERROR OF THE MEAN (σ_M), FROM RAW SCORES OF THE 1963-1964 FINAL TEST
CONTROL GROUP

Raw scores (X)	Frequency (f)	fX	fX^2
135	1	135	18225
126	1	126	15876
117	1	117	13689
108	1	108	11664
105	1	105	11025
99	1	99	9801
96	1	96	9216
95	1	95	9025
93	1	93	8649
88	1	88	7744
87	1	87	7569
85	1	85	7225
83	1	83	6889
79	1	79	6241
77	1	77	5929
75	2	150	11250
74	2	148	10952
73	1	73	5329
71	2	142	10082
70	1	70	4900
68	1	68	4624
66	1	66	4356
65	1	65	4225
63	1	63	3969
57	1	57	3249
56	1	56	3136
54	1	54	2916
53	1	53	2809
52	1	52	2704
45	1	45	2025
42	1	42	1764
30	1	30	900
Total	35	2707	227957

COMPUTATION OF THE MEAN (M):

$$M = \frac{E(fX)}{N} = \frac{2707}{35} = 77.34$$

COMPUTATION OF THE STANDARD DEVIATION (σ):

$$\sigma = \sqrt{\frac{E(fX^2)}{N} - M^2} = \sqrt{\frac{227957}{35} - (77.34)^2}$$

$$= \sqrt{6513.06 - 5981.48} = \sqrt{531.58}$$

$$= 23.05$$

COMPUTATION OF THE STANDARD ERROR OF THE MEAN (σ_M):

$$\sigma_M = \frac{\sigma}{\sqrt{N - 1}} = \frac{23.05}{\sqrt{34}} = \frac{23.05}{5.83}$$

$$= 3.95$$

TABLE X

CALCULATION OF THE t-RATIO
1963-1964 FINAL TEST

Test group	Number of cases (N)	Mean (M)	Standard Deviation (σ)	Standard error of the mean (σ_M)
Experimental	35	90.11	24.08	4.13
Control	35	77.34	23.05	3.95

COMPUTATION OF THE STANDARD ERROR OF A DIFFERENCE (σ_{diff}):

$$\sigma_{diff} = \sqrt{\sigma_{M_E}^2 + \sigma_{M_E}^2}$$

$$= \sqrt{(4.13)^2 + (3.95)^2}$$

$$= \sqrt{17.0569 + 15.6025}$$

$$= \sqrt{32.6594}$$

$$= 5.71$$

COMPUTATIONS OF THE DEGREES OF FREEDOM AND THE t-RATIO

$$D.F. = N_E + N_C - 2 = 35 + 35 - 2 = 68$$

$$t = \frac{M_E - M_C}{\sigma_{diff}} = \frac{90.11 - 77.34}{5.71} = \frac{12.77}{5.71}$$

$$= 2.23$$, significant for 68 D.F.

of a difference, the degrees of freedom and the *t*-ratio. The *t*-ratio is a quotient of the difference between the means of the experimental and the control group (12.77) and the standard error of a difference (5.71) was found to have a value of 2.23. To test whether such a ratio is significant for 68 degrees of freedom, Table C of O. L. Lacey was used.[2]

Significant implication of the t-ratio. As shown in Lacey's table,[3] a *t* of 1.9968 (by interpolation) is needed for the difference in means to achieve a significance at the 5 per cent level of confidence at 68 degrees of freedom. The obtained values of *t* were 2.11 for the first semester of 1962-1963 (Table VII)[4] and 2.23 for the first semester of 1963-1964 (Table X).[5] Since the corresponding values of *t* are greater than 1.9968 which is the value required for the 5 per cent level of confidence at 68 degrees of freedom, the *null hypothesis* is rejected; the two different variables in question, the teaching methods, produced significant differences in performances, and the integrated pattern of teaching is proved to be more effective than the traditional pattern of teaching mathematics.

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[2] *Infra*, pp. 100-101.

[3] *Ibid*.

[4] *Supra*, p. 67.

[5] *Ibid*., p. 72.

CHAPTER V

CONCLUSION AND RECOMMENDATIONS

I. CONCLUSION

First of all, the limitations of this experiment in point of time and scope are fully recognized. It is strongly believed that, to establish the effectiveness of the integrated teaching of mathematics, a much more extensive experimental program on a much longer duration of time should be conducted. But within the framework of this project, and subject to the circumstantial limitations, the following conclusions may be drawn out of the findings:

1. That the integrated teaching of mathematics (geometry and algebra) can give the students a broader perspective of the science of mathematics.

2. That this pattern of instruction can increase the students' ability to correlate the different areas of the science thereby increasing their power to use it as a tool in industry and in their daily living.

3. That because of the first two observations, integrated teaching can be more motivating and challenging.

4. That integrated teaching opens more opportunity for repetition of basic principles, thus reducing to a minimum the element of forgetfulness. The tendency to forget is the

excuse that the students often give when they fail to analyze problems in higher mathematics.

5. Finally, that the adoption of this pattern and technique of teaching mathematics will upgrade mathematics instruction.

II. RECOMMENDATIONS

On the basis of the observations and findings made in this experiment, the researcher submits the following recommendations:

1. That the integrated pattern of teaching the basic concepts of algebra and geometry be adopted in the first year of high school. Within the scope of the experiment, the researcher is quite positive that it will be very effective in laying down the foundation for advanced studies in mathematics.

2. That a continuing experimental program be undertaken by institutions of higher learning, particularly the teacher training colleges, for the purpose of studying further the merits of this pattern of instruction. The findings will be more conclusive and reliable if the experiment is pursued for a period of two or three years using the same students, that is, if the integrated pattern of teaching follows the students as they go up to the higher grades.

3. That a study committee be created to prepare outlines, syllabi, and other materials for the type and pattern

of teaching in this experiment. There are at present no such materials, and unless one is written, mathematics teachers will find it difficult to introduce this new pattern.

4. Lastly, it is strongly recommended that Central Philippine University, where this experiment was conducted, initiate a movement to implement the foregoing recommendations.

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A P P E N D I C E S

APPENDIX A

CENTRAL PHILIPPINE UNIVERSITY
College of Engineering
Jaro, Iloilo City

SURVEY TEST IN MATHEMATICS

GENERAL DIRECTIONS: This is a test to find out how much you know in MATHEMATICS. The questions and problems are distributed in the manner shown in the table below. Read the directions carefully and try to answer as many questions and problems as you can correctly. DO YOUR BEST. (Time Guide--2 hours).

Test		Items		Highest possible score	
Part A. Arithmetic			10		20
Test I		5		10	
Test II		5		10	
Part B. Geometry			25		30
Test I		10		10	
Test II		10		10	
Test III		5		10	
Part C. Algebra			33		70
Test I		15		15	
Test II		10		30	
Test III		4		12	
Test IV		4		13	
TOTAL			68		120

APPENDIX A (continued)

PART A

ARITHMETIC

Test I
DIRECTIONS: Perform the following indicated operations. Indicate your solution in your ANSWER SHEET.

1. Add:
   a. 427.63; 61.4; 0.009; 1.8205
   b. 1 1/2; 3/5; 2 5/6
2. Subtract:
   a. 43.0002 from 692.55
   b. 13 2/3 from 19 3/8
3. Multiply:
   a. 5.42 by 3.06
   b. 11 1/2 by 7 1/2
4. Divide:
   a. 611.82 by 8.24
   b. 6 by 3/4
5. Extract the square root:
   a. 7.84
   b. 25.0000

APPENDIX A (continued)

Test II.
DIRECTIONS: Give a complete solution of each of the following problems in your ANSWER SHEET.

1. A man worked from 8:00 A.M. to 12 noon and from 1:00 P.M. to 4:00 P.M. at ₱0.48 an hour. How much did he earn for the day?

2. Find the simple ordinary interest at 6% per annum on ₱20.00 for 36 days.

3. If oranges are bought at the rate of 2 for 5 centavos and sold at the rate of 3 for 10 centavos, find the profit on 60 oranges.

4. A man was born on June 12, 1934. Find his present age in years, months, and days.

5. To be in the honor roll Mary has to have an average of 85 in all her work. In three subjects, she received 83, 85, and 80. What mark must she receive in her fourth subject to make the honor roll?

PART B
GEOMETRY

Test I.
DIRECTIONS: Draw in freehand the figure of each of the following in your ANSWER SHEET.

1. An acute angle
2. A square
3. An isosceles triangle
4. A right triangle
5. A hexagon
6. A trapezoid
7. Two concentric circles

APPENDIX A (continued)

8. A circle inscribed in a square

9. A circle circumscribed about a triangle

10. A tangent

Test II.
DIRECTIONS: Fill the blanks with the word that will satisfy the statement. Use your ANSWER SHEET.

1. Two intersecting lines that form right angles are called __________ lines.

2. The space or opening between two lines that meet is called a/an ________.

3. A _______ is any plane figure bounded by four straight lines.

4. A _______ is any plane figure bounded by three straight lines.

5. An angle having an opening less than 90° is called a/an _____ angle.

6. An angle that is greater than 90° but less than 180° is called a/an ______ angle.

7. An angle of 180° is called a/an ______ angle.

8. A straight line from the center of a circle to the circumference is known as the ___ of the circle.

9. Two angles whose sum is 90° are called ___ angles.

10. The side of a right triangle opposite the right angle is called the _______.

Test III.
DIRECTIONS: Give a complete solution of each of the following problems in your ANSWER SHEET.

1. Find the base of a right triangle if the height is 5 feet and the hypotenuse is 13 feet.

APPENDIX A (continued)

2. Over what area can a goat graze if it is tied to a stake with a rope 10 feet long?

3. A triangle has one angle equal to 30°. Another angle is equal to 45°. Find the value of the third angle.

4. A rectangle has an area of 39 square meters. Its base is 13 meters. Find its perimeter.

5. At the same time that the flagpole 60 feet high casts a shadow 40 feet long, a high building casts a shadow of 90 feet long. How high is the building?

PART C

ALGEBRA

Test I.
DIRECTIONS: Express each of the following statements in terms of algebraic symbols. Use your ANSWER SHEET.

1. A number x added to y.
2. A number m diminished by n.
3. The square of a number a.
4. The square root of a number x.
5. Twice a number x.
6. Half a number y.
7. Three greater than x.
8. The difference between the digits of the number 35.
9. D exceeds x.
10. A number x increased by half of itself.
11. A boy is 12 years old now. How old was he m years ago?

12. In the preceding question, how old will the boy be <u>x</u> years hence?

13. Express the perimeter of a rectangle one side of which is <u>x</u> and the other side three times as long.

14. A number <u>x</u> varies directly as <u>y</u>. If <u>x</u> is doubled, what happens to <u>y</u>?

15. A number <u>m</u> varies inversely as <u>n</u>. If <u>n</u> is halved what happens to <u>m</u>?

Test II.
DIRECTIONS: Perform the indicated operations and simplify. Use your ANSWER SHEET.

1. Add:

   a. 18.4 and -72

   b. $\frac{4x}{y}$; x - 2y ; -5x - y

   c. $3\sqrt{3} + 2\sqrt{3} + 5\sqrt{5}$

   d. x ; x^2 ; x^3

2. Subtract:

   a. -56 from 11

   b. ax from my

   c. (x - 2y) from $\frac{4x}{y}$

3. Multiply:

   a. $(a^2)(a)(a^3)$

   b. $-x(m - \frac{y}{2})$

   c. $(\sqrt{8})(2\sqrt{2})$

4. Divide:

   a. $9a$ by 3

   b. 2 by $\sqrt{2}$

   c. $72ax^4y$ by $36a^2x^3$

5. Remove the sign of grouping and combine like terms:

$$x - 2[y + 3 - (a - 2b)]$$

6. Factor the following expressions:

   a. $a^2 - b^2$

   b. $x^2 - 2x - xy$

   c. $x^2 - x - 12$

7. Find the highest common factor for the following expressions:

   a. 6, 12, 18, 24

   b. $(a - b)^2$, $(a - b)$, $(a^2 - b^2)$

   c. $(x^2 - 2x)$, $(x^2 - 4)$, $(x^2 - 4x + 4)$

8. Reduce the following fractions to lowest terms:

   a. $\dfrac{4a^2 - 8a}{2(a - 2)(a + 2)}$

   b. $\dfrac{\frac{1}{a} + \frac{1}{b}}{1 + \frac{1}{b}}$

9. Solve for the value of x:

   a. $15x + 10 = 3x + 14$

   b. $x^2 - 16 = 0$

   c. $\dfrac{3x - 5}{2x + 10} = \dfrac{2}{3}$

10. Solve simultaneously for the unknown:

$$x + y - 15$$
$$2x - y - 6$$

Test III.
DIRECTIONS: Using only ONE VARIABLE, express the following facts as algebraic equations. Do not solve. Use your ANSWER SHEET.

1. The sum of two numbers is 120. If 2/3 of one is 12 greater than 3/4 of the other, what are the numbers?

2. At what time between 9 and 10 o'clock will the hands of the clock be opposite each other?

3. In 8 years a boy will be twice as old as he was 4 years ago. How old is the boy now?

4. How many pounds of a 30-centavo candy must be mixed with 50 pounds of a 35-centavo candy to make a mixture of a 33-centavo candy?

Test IV.
DIRECTIONS: Using TWO VARIABLES, make an algebraic equation for each of the following problems. Specify your assignment of the unknowns. Do not solve. Use your ANSWER SHEET.

1. The sum of two numbers is 50 and their difference is 12. Find the numbers.

2. If A and B together have 40 pesos, B and C have 50 pesos, A and C have 60 pesos, how much has each?

3. The sum of three fractions is 4. The first fraction is double the second and the second double the third. What are the fractions?

4. The length of a rectangle is one foot greater than the width. If the dimensions are increased by one foot, the area would be increased by 30 square feet. What are the dimensions?

APPENDIX B

CENTRAL PHILIPPINE UNIVERSITY
College of Engineering
Jaro, Iloilo City

FINAL TEST IN MATHEMATICS

GENERAL DIRECTIONS: This is a test to determine how much you have learned in MATHEMATICS during the semester. The exercises and problems are distributed in the manner shown in the table below. Read the directions carefully and try to answer as many exercises and problems as you can correctly. DO YOUR BEST. (Time guide--2 hours).

Test	Items	Highest possible score
Test I	10	50
Test II	6	24
Test III	6	30
Test IV	8	56
TOTAL	30	160

APPENDIX B (continued)

Test I.

DIRECTIONS: A. Perform the following indicated operations and simplify. Indicate your solution in your ANSWER SHEET.

1. What expression must be added to a - 3b - c to produce an expression equal to the sum of 2a - 3c and c - 3b - a?

2. $\frac{x^2 - y^2}{2ax + 2ay} \cdot \frac{ax}{2x - y} \div \frac{x^2 - xy}{4ax - 2ay}$

3. $8^{2/3} + 25^{-1/2} - 64^{1/6} + \frac{1}{4^{-1}}$

4. $2x\sqrt{72} - 5x\sqrt{98} - 16x\sqrt{50}$

5. $\frac{1 + \frac{y}{x - y}}{1 - \frac{y}{x + y}}$

B. Solve for the value of the unknown or unknowns:

6. $3x + 5y = -7$
   $2x - y = 4$

7. $\frac{2x + 4}{x - 3} = \frac{x}{2} - \frac{3x - 2}{6}$

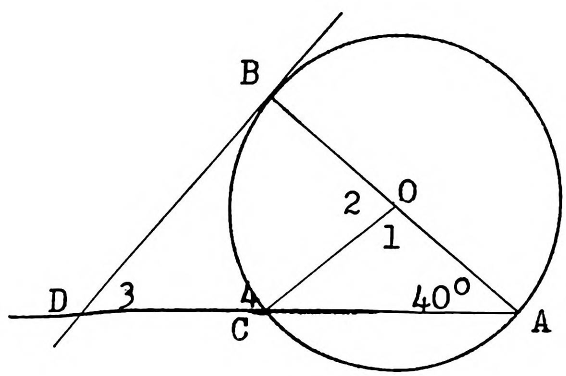
8. $\sqrt{4x + 1} = x - 5$

APPENDIX B (continued)

C. Indicate your solution for the required values in your ANSWER SHEET.

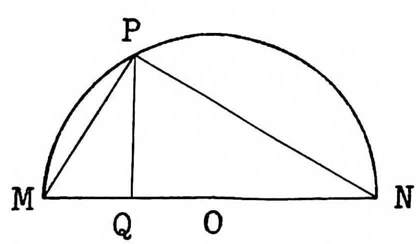
9. Given: A circle with O as center, AB as diameter, OC as radius, BD as tangent, and angle OAC equal to 40 degrees.

Required: Find the values of angles 1, 2, 3, and 4, in degrees.



10. Given: O is the center of the circle. PQ is perpendicular to MN.

Required: Make 3 possible proportions.



APPENDIX B (continued)

Test II.
DIRECTIONS: Express each of the following statements in terms of algebraic symbols. Use your ANSWER SHEET.

1. The larger of two weights is four pounds less than twice the smaller. How heavy is the larger weight if the smaller weighs $\underline{x}$ pounds?

2. The larger of two numbers is four more than the smaller. If the larger is $\underline{x}$, what is the smaller?

3. What number exceeds $\underline{y}$ by four less than $\underline{x}$?

4. What was John's age four years ago, if he will be $\underline{y}$ years old in five years?

5. The altitude of a rectangle is $\underline{h}$ units long. The base is three units longer than the altitude. What is the area of the rectangle?

6. A has $\underline{x}$ more marbles than B, and B has twice as many marbles as C. If C has $\underline{m}$ marbles, how many marbles has A?

Test III.
DIRECTIONS: DO NOT SOLVE. Write only your Representation and Equation in your ANSWER SHEET.

1. George's age in 8 years will be four times what it was last year. How old is George?

2. At what time between 10 and 11 o'clock are the hands of a clock opposite each other?

3. A can do a piece of work in 1 1/2 days. B can do it in 1/2 day. If they work together, they can finish the work in how many days?

4. Find three consecutive even numbers such that the sum of the squares of the smaller two equals the square of the largest number.

5. The length of a rectangle is 5 more than its width. If its width were doubled and its length were halved, the perimeter of the rectangle would be increased by 12. Find the dimensions of the rectangle.

APPENDIX B (continued)

6. A motorboat capable of traveling 10 miles per hour in still water takes 30 minutes longer to travel upstream a distance of 12 miles than it takes to travel downstream the same distance. Find the rate of the current.

Test IV.

DIRECTIONS: Give a complete solution of each of the following problems in your ANSWER SHEET.

1. A tower 100 feet high is situated on the bank of a river. A Boy Scout on the opposite bank finds that the angle between the line from his eye to the top of the tower and the line from his eye to the foot of the tower is 30°. How wide is the river?

2. If 2 men can plow 6 acres of land in 4 hours, how many men are needed to plow 18 acres in 8 hours?

3. A purse contains \$2.10 in nickels and dimes. If there are 29 coins in all, how many of each kind are there?

4. Two parallel lines are cut off by a transversal. If one of the two interior angles on the same side of the transversal is three times the other, find the number of degrees in the larger angle.

5. The diagonal of a cube is $8\sqrt{3}$ inches. Find the volume of the cube.

6. Wagwag rice costs ₱25 a cavan; NARIC rice, ₱17.50. How many cavans of wagwag rice should be added to 8 cavans of NARIC rice to make a mixture that would cost ₱22 a cavan?

7. The base of a triangle is 15 inches and its area is 60 square inches. Find the area of a similar triangle whose altitude is 6 inches.

8. The base of the oblique cone has a diameter of 14 inches. Find the volume of the cone if the axis is 16 inches and is inclined to the base at an angle of 60°.

APPENDIX C

COURSE OUTLINE

Mathematics 110

REVIEW OF HIGH SCHOOL MATHEMATICS

AND SOLID GEOMETRY

TEXTBOOKS: Edgerton and Carpenter: Intermediate Algebra
Strader and Rhoads: Plane Geometry
Wentworth and Smith: Solid Geometry

PART I

FUNDAMENTAL CONCEPTS
(2 weeks)

Week

I THE HISTORY AND DEVELOPMENT OF THE USE OF NUMBERS

*Real Numbers
*The Limited and the Unlimited Number Systems
*Signed Numbers
*Rational and Irrational Numbers
*Fractions and Decimals

II THE FUNDAMENTAL OPERATIONS

*Addition and Subtraction
*Multiplication and Division
*Involution and Evolution

PART II

ARITHMETIC AND ALGEBRA
(6 weeks)

I PRODUCTS AND FACTORS
(These are taught simultaneously, i. e., after studying a certain type of product, the reversed process of factoring is taken)

* Indicates where integration was made.

APPENDIX C (continued)

*Product of Monomials and Polynomials
*Square of the Sum and Difference of a Binomial
*Product of Sum of Two Quantities by their Difference
*Product of Binomials Having Similar Terms
*The Binomial Theorem
Other Types of Factoring

II FRACTIONS

*H.C.F. and L.C.M. and their Applications
Fractional Operations
Complex Fractions

III POWERS AND ROOTS

EXPONENTS

The Zero Exponent
The Negative Exponent
The Fractional Exponent
Application of Exponents in Equations

RADICALS

Transformation of Radicals
Addition and Subtraction of Radicals
Multiplication and Division of Radicals
Application of Radicals in Equations

IV IMAGINARIES

*Vector Representation
*Complex Numbers
*Addition and Subtraction of Complex Quantities
*Multiplication and Division of Complex Quantities

V EQUATIONS

*Linear Equations (With Graphical Representation)
*Quadratic Equations (With Graphical Representation)
*Simultaneous Systems (With Graphical Representation)
VI *Applications in Problems

APPENDIX C (continued)

PART III

GEOMETRY
(8 weeks)

I <u>THE CONCEPT OF SPACE</u>

Definition of Geometric Terms
Loci and Developments of Geometric Figures

<u>GEOMETRY OF ONE DIMENSION</u>

Lines and their Properties
Mensuration and Applications
II *Angles and their Properties

<u>GEOMETRY OF TWO DIMENSIONS</u>

Triangles and their Properties
Quadrilaterals and their Properties
III Polygons and their Properties
Circles and their Properties
IV *Similarities of Geometric Figures
Mensuration and Applications

V <u>GEOMETRY OF THREE DIMENSIONS</u>

Dihedral and Polyhedral Angles
Solids and their Properties
- Prisms
- Parallelepipeds
- VI Pyramids
- Cylinders
- Cones
- VII Spheres
- Measurement of Spherical Surfaces
- Measurement of Spherical Solids
- VIII Application in Problems

TABLE C*

VALUES OF t AT THE 5% AND 1% LEVELS OF SIGNIFICANCE

Degrees of Freedom	5%	1%
1	12.706	63.657
2	4.303	9.925
3	3.182	5.841
4	2.776	4.604
5	2.571	4.032
6	2.447	3.707
7	2.365	3.499
8	2.306	3.355
9	2.262	3.250
10	2.228	3.169
11	2.201	3.106
12	2.179	3.055
13	2.160	3.012
14	2.145	2.977
15	2.131	2.947
16	2.120	2.921
17	2.110	2.898
18	2.101	2.878
19	2.093	2.861
20	2.086	2.845
21	2.080	2.831
22	2.074	2.819
23	2.069	2.807
24	2.064	2.797
25	2.060	2.787
26	2.056	2.779
27	2.052	2.771
28	2.048	2.763
29	2.045	2.756
30	2.042	2.750
60	2.000	2.660
100	1.984	2.626
200	1.972	2.601
500	1.965	2.568
1000	1.962	2.581
	1.95996	2.57582

* This table is a portion of Table C of O. L. Lacey, *Statistical Methods in Experimentation: An Introduction* (New York: The Macmillan Company, 1953).

INTERPOLATION OF t AT THE 5% LEVEL OF SIGNIFICANCE:

Degrees of Freedom	5%
60	2.0000
	.0032 (x)
68 (8)	1.9968
100 (40)	1.9840 (.0160)

$$\frac{x}{.0160} = \frac{8}{40}$$

$$x = (.0160)\left(\frac{8}{40}\right)$$

$$x = .0032$$

VALUE OF t FOR 68 D.F. AT THE 5% LEVEL OF SIGNIFICANCE:

$$= 2.0000 - .0032$$

$$= 1.9968$$